Atmospheric Propagation Effects on Geodetic Quality Radio Measurements: Ray Description

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I. Propagation Effects

Geodetic measurements of range or range rate are often made between satellites, such as GPS, and the ground. This discussion will apply to signal paths that traverse all or almost all of the atmosphere. This applies to ground to satellite cases as well as high altitude observations. The signal transverses the atmosphere where it is affected in two ways. Both the effective length and the angle of arrival are changed. Only the effects of the neutral atmosphere will be addressed here. Ionospheric effects are not discussed.

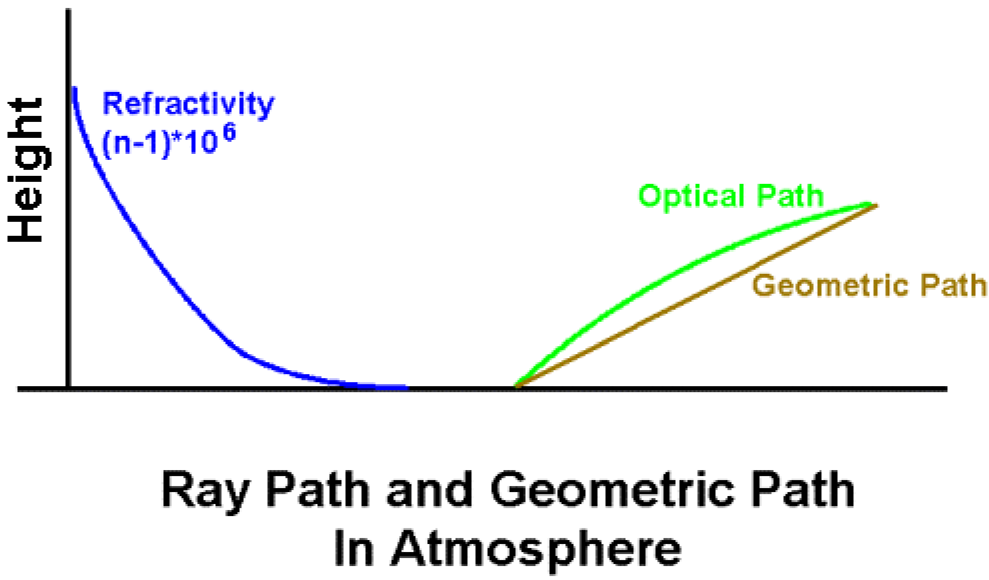

Figure 1

A diagram of a ray path is shown in Figure 1. Here the geometric path is not followed because the refractive index changes along the path. It is the spatial change, that is the gradient in the refractive index, that leads to bending. If the refractive index, n, were constant but not unity, there would be no bending. There would, however, be propagation effects. The effective length would be changed.

The deviation of the refractive index from unity in the atmosphere is often expressed in terms of the refractivity, N.

$$N \quad = \quad 10^6 \ (n-1)$$

The refractivity is scaled to make it a number easier to handle manually. At the surface it is often between 250 and 350 "N-units". In satellite to ground propagation problems, the electron density is also commonly called "N". There should be no confusion because we are not considering the ionosphere. Here "N" is the meteorologist refractivity.

Both the bending and the lengthening are called "refraction" in the literature. In geodetic applications, the delay is often called refraction and the bending ignored. Here we will not use the term refraction to avoid this confusion. The length or integral effects will be called the tropospheric delay. The bending (true refraction) will simply be called bending.

II. Length or Integrated Effects

The measured path length, ρ_m, will be the integral of the refractive index along the path of the ray.

$$\rho_m = \int_{Optical\ Path} n \ ds$$

This integral can be broken apart two ways. First the n is replaced by the sum of 1 and (n-1). This will lead to the introduction of N in the equations. Second the path is broken into the geometric path, and the difference between the two paths. Ignoring terms second order in smallness,

$$\rho_m \quad = \quad \int_g 1 \ ds + \int_o (n-1) \ ds + \left[\int_o 1 \ ds - \int_g 1 \ ds \right]$$

Where the path g is the geometric path and the path o is the optical path. The first term is the true length, the geometric length,

$$\rho_0 = \int_g 1 \ ds$$

The next term represents a lengthening of the path, or a delay in the signal arrival in units of length. In the GPS literature it is called the tropospheric delay, but commonly expressed in length units.

$$\rho_t = \int_o (n-1)\, ds$$
$$= 10^{-6} \int_o N\, ds$$

For a typical atmosphere, the tropospheric delay, ρ_t, is about 2.5 m in the zenith direction. At lower elevation angles it increases. Very near the horizon it can be over 50 m.

The last term is usually small because the deviation from the geometric path is slight. It is

$$\Delta = \left[\int_o 1\, ds - \int_g 1\, ds \right]$$

Note that for the case of ducting, this correction can be extremely large. This term is generally ignored in GPS work. This is acceptable if measurements are only used above elevation angles of 10 degrees or more.

III. Bending

The atmosphere will change the angle of arrival of the signal. This can be computed from Snell's law or by solving the equations for the electromagnetic wave. Here Snell's law, in one of its forms, will be used. This implies that there are no significant variations in the refractivity on the order of a wavelength. If that were the case, the parabolic equation approximation to Maxwell's equation would likely be adequate.

There are three common statements or approximations used for Snell's law. The first assumes that the refractive index is constant except at boundaries where it changes discontinuously to a new value. This will be called the Boundary Model. The second also assumes that the atmosphere consists of layers. In this case the gradient of the refractive index is assumed constant within a layer, and it changes discontinuously at the layer boundaries. This will be called the Layer or Constant Gradient Model.

The third statement of Snell's law is called "Spherical Snell's" law. It applies to shells about a common origin. The refractive index gradient is assumed constant within each shell. This is a variant of the Layer or Constant Gradient Model.

1. Boundary Model

The boundary version of Snell's law is usually stated in terms of the angle of incidence of the rays with respect to the boundary. However for propagation work it is more convenient to use the elevation angle, ξ, which is the complement of the angle of incidence. Using elevation angle, as shown in Figure 2, Snell's law becomes:

$$n_0 \cos(\xi_0) \quad = \quad n_1 \cos(\xi_1)$$

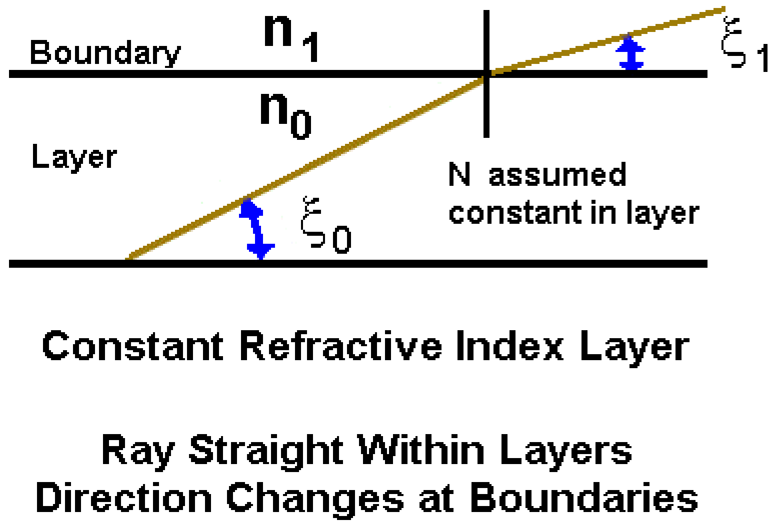

Figure 2

2. Constant Gradient Model

If the gradient of the refractive index is constant within the layer as in Figure 3, the ray follows a curved path. For very shallow gradients, it can be shown that this path is part of a circle. The radius of this circle is called the radius of curvature of the ray, R_c. In fact the radius of curvature can be defined continuously along the path, leading to a more general description of the path. The inverse of this, R_c^{-1} is usually found in terms of the gradient.

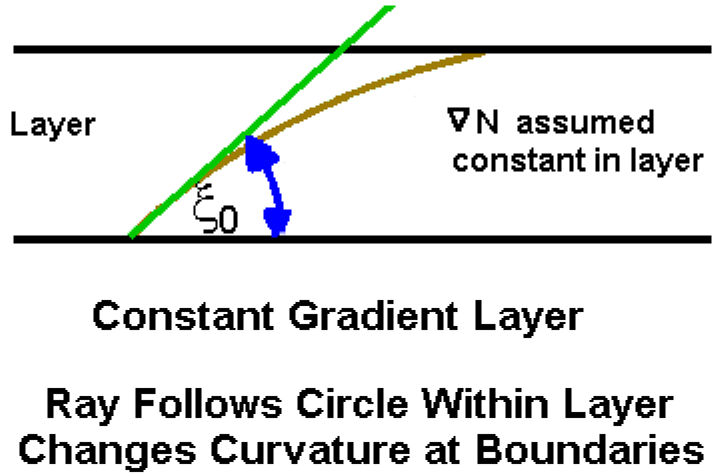

Figure 3

In order to find the radius of curvature equation consider the infinitesimal layer shown in Figure 4. The orientation of the layer is determined by the direction of the gradient of n, ∇n. Here the gradient is down, to correspond to the atmospheric case
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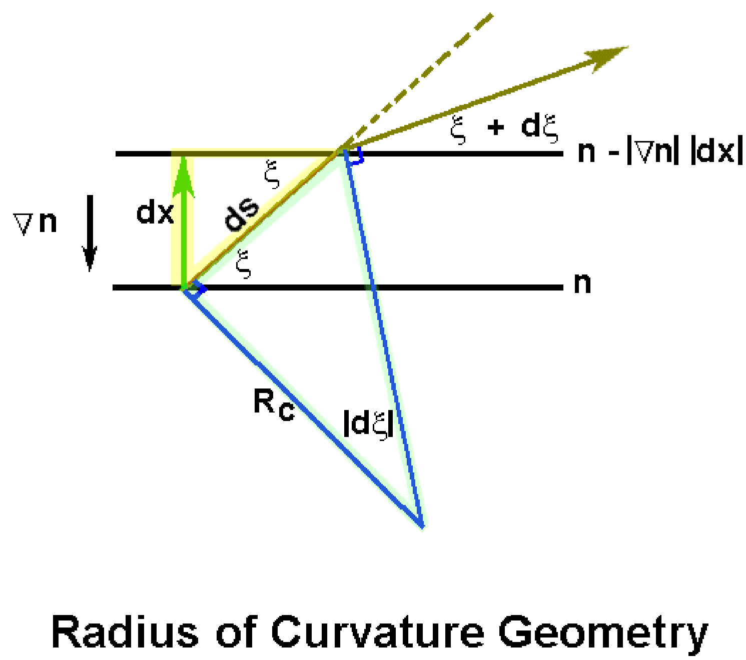

Figure 4

The approach is to relate ds, the ray step, to the angle change dξ and hence the radius of curvature R_c,

$$\begin{aligned} ds &= R_c \left| d\xi \right| \\ R_c &= \frac{ds}{d\xi} \end{aligned}$$

The value of dξ is determined from Snell's law that can be stated as

$$n \cos(\xi) = \text{constant} .$$

Taking the differential,

$$\begin{aligned} dn \cos(\xi) - n \sin(\xi)\, d\xi &= 0 \\ \nabla n\, dx \cos(\xi) &= n \sin(\xi) \end{aligned}$$

From the upper highlighted triangle, one has,

$$\frac{dx}{ds} = \sin(\xi)$$

and therefore

$$\begin{aligned} \nabla n\, dx \cos(\xi) &= n \frac{dx}{ds}\, d\xi \\ \nabla n \cos(\xi) &= n \frac{d\xi}{ds} \\ &= n\, R_c^{-1} \end{aligned}$$

The quantity R_c^{-1} is also called the curvature. It is finite even when the line is straight. From the above we obtain

$$R_c^{-1} = \frac{1}{n} \nabla n \cos(\xi)$$

the desired equation for the radius of curvature.

In some meteorology books the cos (ξ) factor is left off. This is done because the propagation is assumed to be approximately horizontal and the gradient of n in the atmosphere is essentially in the vertical direction.

3. Spherical Snell's Law

For the case of a set of concentric shells, the Constant Gradient Model leads to the statement that $nr\cos(\xi)$ is constant. In the case of the refractive index being identically one, this must lead to a straight line. This is seen from the left side of Figure 5.

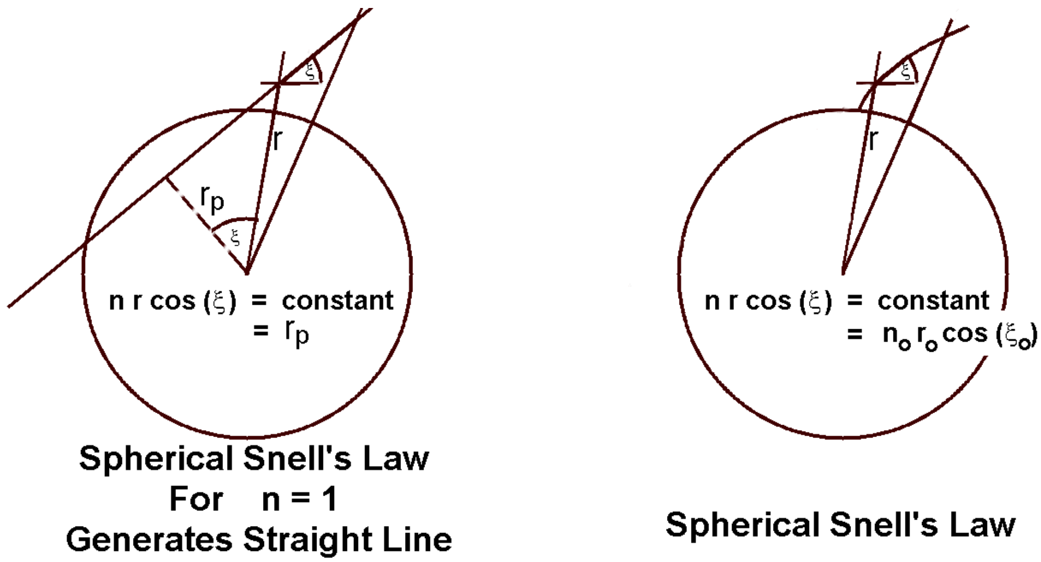

Figure 5

Now in the case of a real refractive index, the ray will be curved, but Spherical Snell's law still applies. Now the constant is cannot be interpreted as easily. It is usually just given as the value of n r cos (ξ) at the ground. Above the atmosphere n is just 1, thus there

$$r\cos(\xi) \quad = \quad n_0\, r_0 \cos(\xi_0)$$

It seems that above the atmosphere, the elevation angle is only a function of the ground conditions and current altitude. The shape of the profile doesn't seem to matter. This is not the case.

The details of the profile determine the amount that the ray is translated off the geometric ray. The elevation angle is measured from the local horizon – determined by the line from the center of the earth. Therefore two lines at the same altitude with the same elevation angle, but different locations, will point to different places in space. In the extreme case of a duct, this can be very large as diagramed in Figure 6.

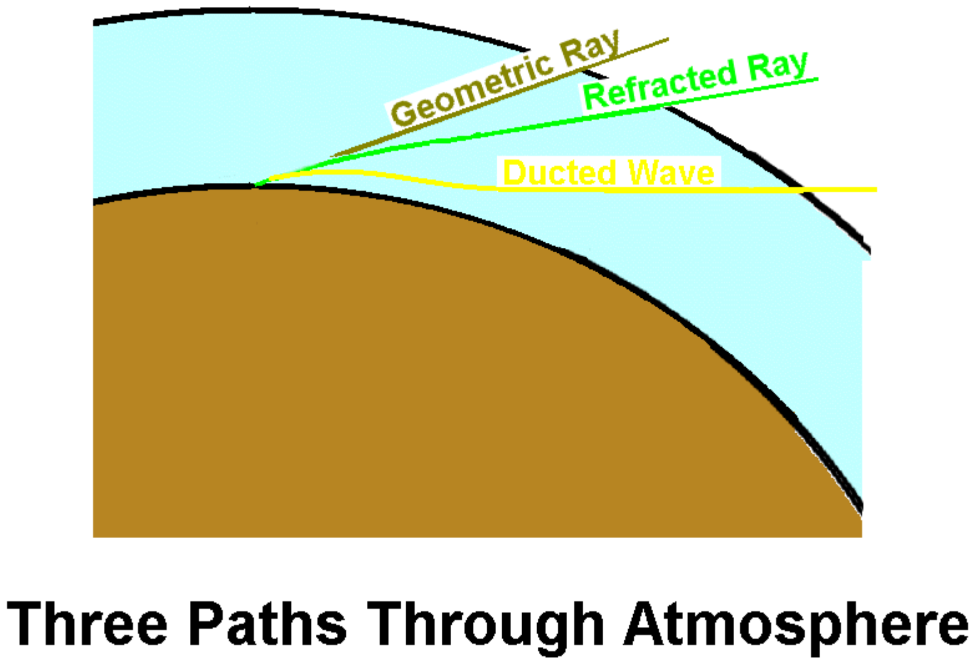

Figure 6

4. Real Atmosphere Geometry

A diagram of a ray, including the lines to the center of the earth, is shown in Figure 7. The ray leaves the ground at an elevation angle of ξ_{go}. There are two useful elevation angles at an altitude above the atmosphere. The line that leaves the ground at the initial elevation angle has an elevation angle, ξ_{ao} at altitude. The second elevation angle, ξ_{a}, is formed by the true ray.

Another important angle is the one measured from the center of the earth. This is called the earth central angle, θ. The central angle formed by the straight line and the true ray are different. This difference will directly affect the elevation angles.

Above the atmosphere both the geometric path and the ray path are straight lines but going in different directions. The angle between these lines will be the sum of all the bending, called $d\xi$ here. (In some meteorology literature this is called τ, but that is often used for delay in the geodetic literature.) This angle is seen at the intersection of the ground straight line and the straight backward extension of the true path.

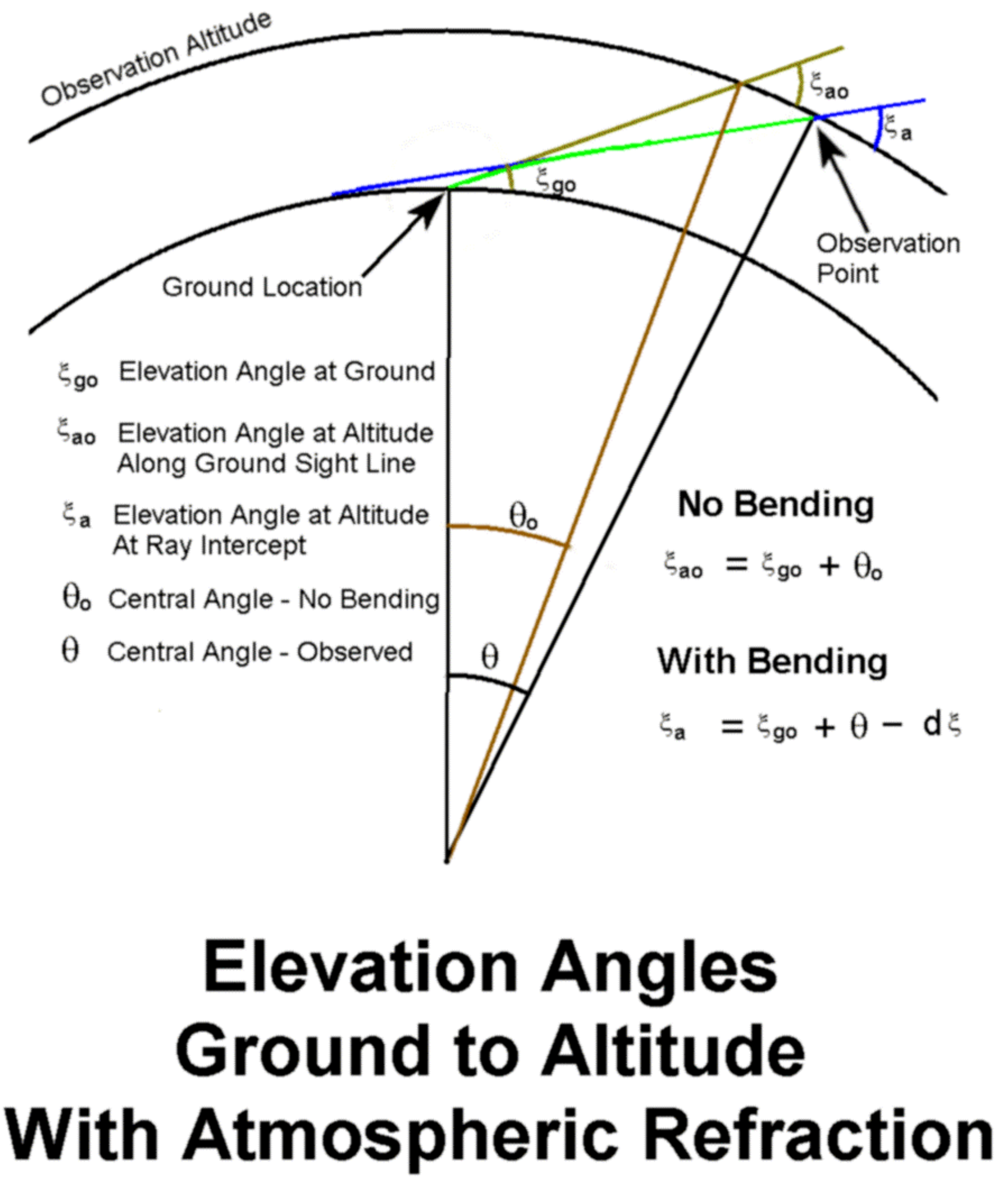

Figure 7

The elevation angle at altitude is different from the ground value for two reasons. First the horizontal reference line has changed as the direction of "up" changes. In this spherical earth model the line from the center of the earth defines this direction. In fact the change is a simple addition for the geometric ray.

$$\xi_{ao} = \xi_{go} + \theta_0$$

For the real ray there will be two changes. First the intercept point at the same altitude will be different. This will cause a change in the earth central angle

$$\theta = \theta_0 + d\theta$$

Secondly, there will have been some true bending of the ray. Defining bending as positive downward, we have

$$\begin{aligned} \xi_a &= \xi_{go} + \theta - d\xi \\ &= \xi_{go} + \theta_0 + d\theta - d\xi \\ &= \xi_{ao} + d\theta - d\xi \end{aligned}$$

Now both dθ and dξ are dependent on the exact shape of the profile. If all the change in refractive index occurs at or very near the ground, then the bending will occur there. The turned ray will act over a large distance and dθ will be largest. If the refraction change occurs very near the observation altitude the bending angle will have little or no distance over which to act.

The bending would be the same in both these cases for a planar system. For a spherical case the gradient of the refractive index is position dependent. It is assumed to point down. Thus dξ is also a function of the shape. For the real atmosphere where most bending occurs near the lowest altitude, dξ is only weakly dependent on the shape - except for rays very close to horizontal. This case is important because it describes a large fraction for radar data.