Ionospheric Bending of Radio Signals: A Rough, Shell Model Approximation

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I. Introduction

A rough estimate of the bending of radio waves that transit the ionosphere is computed. This is done with a simple one-layer shell model. The parameters of the shell are varied to match known extremes of ionospheric conditions. Computations were performed from 150 MHz to 5 GHz.

The bending is found to vary as $1/f^2$, where f is the operating frequency. This agrees with a simpler analytic computation. A numeric simulation was done for a ground elevation of zero degrees (horizontal at the ground). The results of three cases, spanning the range of parameters, are presented. The bending was between 14 and 4 mrad at 150 MHz.

In the following sections some background material on the ionosphere is given to support the use of a shell model. The choice of shell parameters also comes from material in this section. The refraction of radio waves is very briefly discussed and the Spherical Snell's Law introduced and interpreted. The relation between the horizontal offset and the bending is shown. This is then used to generate numeric results reported in the last section.

II. The Ionosphere

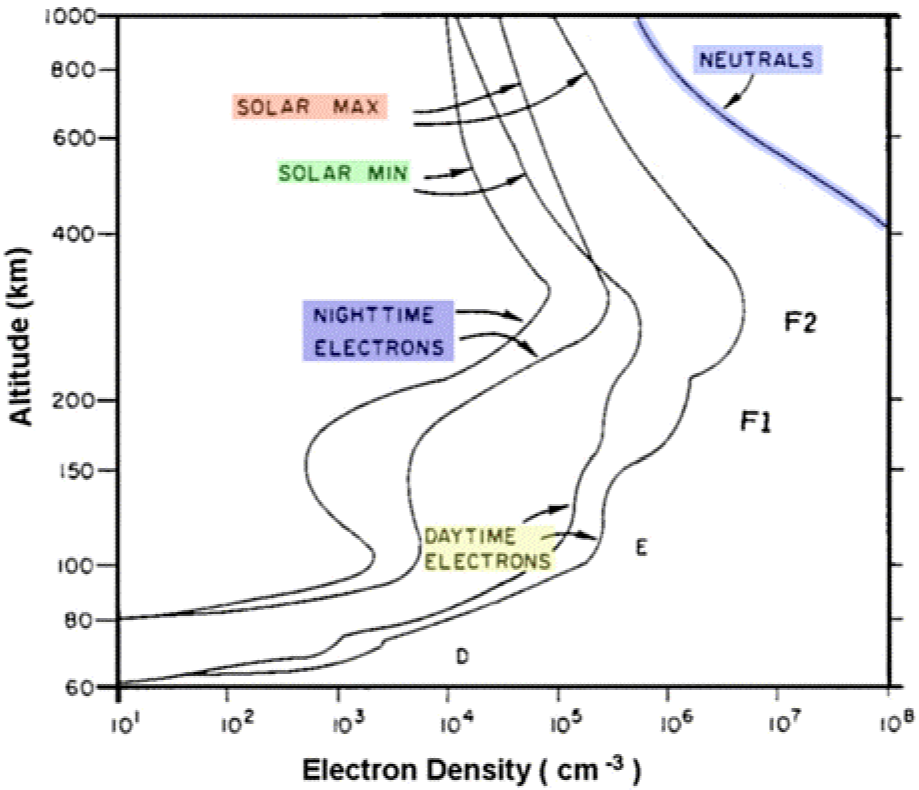
The ionosphere is a region, just above the atmosphere, were there are a significant number of free electrons and ions. The number of ions and electrons is equal, so there is no net charge. This is called a plasma. The electrons and ions respond to the electric fields of passing radio waves. This causes the refractive index to be modified, and radio waves are slowed and their direction changed. Because of the very large mass difference between the electron and the ions, the electrons have the largest effect. This is particularly true at frequencies above a MHz. The effects are proportional to the number density of electrons, the number of electrons per cubic meter.

The ionosphere is generated by the extreme ultraviolet and x-ray emissions of the sun. These do not come from the same region that we see with our eyes. This radiation is a very small fraction of the solar emission, and varies with the 11.5 year solar cycle.

The ionosphere generation involves photochemical reactions – some of a complex nature. The number of electrons and ions is a balance between the solar radiation breaking electrons off neutral particles and the recombination of electron and ions. There are two key items of interest here. First, the recombination rates are much slower than the production rates. This leads to the persistence of the ionosphere at night. The level is much lower by sunrise however.

The second major feature comes from the separation with height of different neutral particle types. Near the ground the atmosphere is well mixed. At high altitudes the lighter particles rise higher than the heavier particles. This leads to layers in the ionosphere. Different reactions lead to significantly different densities of electrons in the layers.

A generic diagram of the electron density as a function of altitude is shown in Figure 1. There are 5 curves on this busy plot. The y-axis is the height on a logarithmic axis. The x-axis is the number of particles, also a logarithmic axis. The use of a log-log plot distorts the shape. Gentle peaks on the graph are very sharp on a linear plot.



Electron Density Model Profiles
Note Extreme Variations
Log-Log Scale Used

Figure 1

The layers are named with letters based on the historical conventions. The largest layer, the F-region, is now know to be split into two layers called the F1 and F2. For the purpose of the simulations that follow, the F region will be the only one considered. The ionosphere F region will be modeled as a shell of constant electron density.

Four of the lines on Figure 1 represent electron densities. Two variables are separated, day vs. night and solar maximum vs. solar minimum. In general, the day-night changes are most significant. After that, the solar cycle variation is the next largest. On the upper right in Figure 1, the number density of the neutral particles is shown. Note that the high level of neutrals implies that only a very small fraction of the particles are ionized. The ionosphere is a "weakly ionized plasma".

It should be emphasized that these are schematic values. No individual day looks exactly like these plots. The day to day variation can be large, particularly in a solar storm (geomagnetic storm).

One variation not shown here is that of position. Clearly there should be more ionization at the sub-solar point than where the sun is low on the horizon. This same effect causes the temperature on the ground to be generally higher in the tropics than high latitudes. In the case of the ionosphere this is also true, but effects of the earth's magnetic field distort the exact shape.

Contour plots of a NOAA model ionosphere are shown for solar maximum (Figure 2) and solar minimum (Figure 3). These are both for 1800 UT. At this time the sun is rising at 180 degrees longitude, setting at 0 degrees longitude and noon is along 90 W.

The values are given in terms of the peak plasma frequency at the given location. The plasma frequency, f_p, is a measure of the electron density. The electron density is just a constant times the square of f_p. Radio signal below this frequency cannot go through the ionosphere.

Notice the two large humps just north and south of the equator. These are caused by the earth's magnetic field. In fact the peaks are about 12 degrees north and south of the geomagnetic equator. That line is shown on these maps. It dips furthest south over South America and furthest north over Africa. It is slightly irregular, as is the earth's magnetic field.

These humps, or equatorial anomalies, have the highest electron densities. The values from here will be used to get a conservative over estimate of the bending of a radio wave going through the ionosphere.

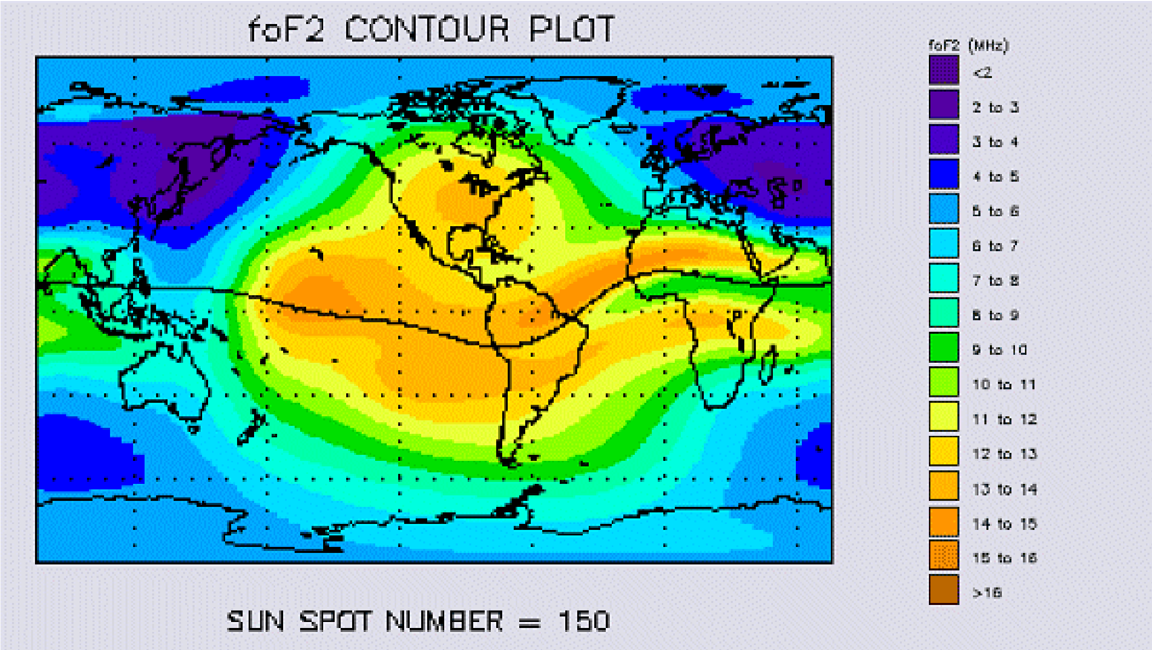


Figure 2

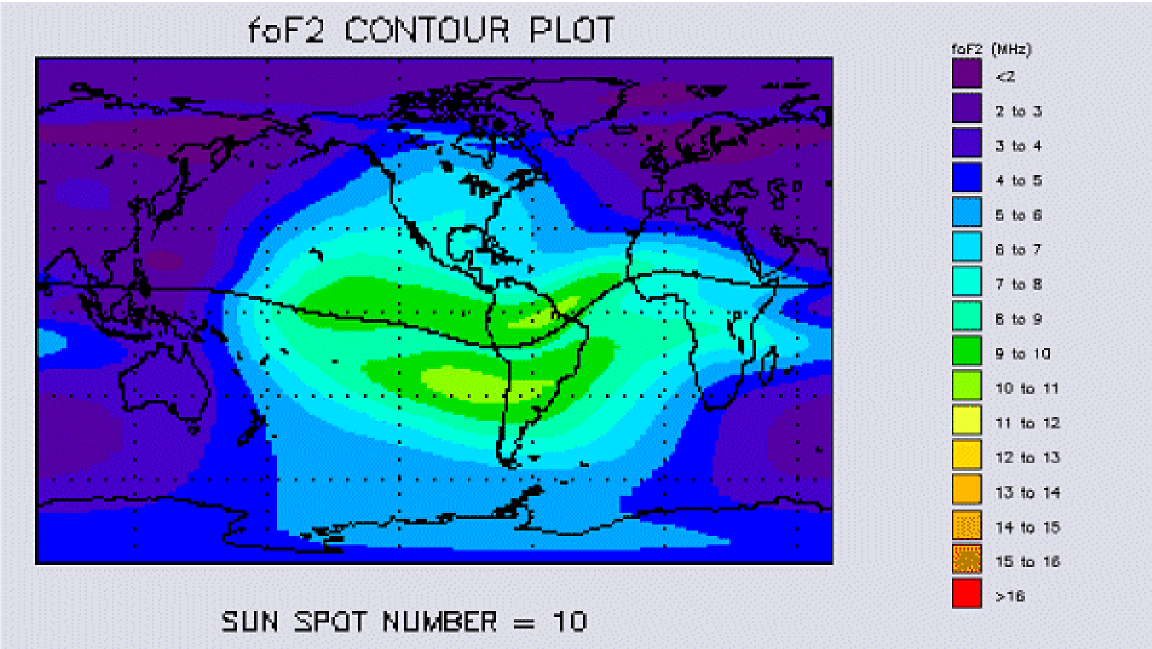


Figure 3

III. Refractive Index at VHF and Above

The generic form of the refractive index of a radio wave in the ionosphere is very complex. It is know as the Appletion-Hartree equation. However at frequencies above a few MHz, it can be simplified to

$$n = \sqrt{1 - \frac{f_p^2}{f^2}} \; ,$$

where n is the refractive index, f_p is the plasma frequency, and f is the frequency of the radio wave. When the value in the square root is negative, the wave cannot propagate. As seen in Figures 2 and 3, the value of f_p is always less than 20 MHz. The equations here are valid for frequencies above 100 MHz and these waves will always propagate through the ionosphere.

IV. Elevation Angles and Spherical Snell's Law

A diagram of a radio wave leaving the ground at an elevation angle of ξ_g and entering the ionosphere at an elevation angle of ξ_a is given in Figure 4. It is important to note that the elevation angles are measured with respect to the local vertical. The line from the center of the earth defines this. Therefore a straight line will have a varying elevation angle along its position. The most important aspect of this concerns the minimum elevation angle in the ionosphere. Because the major ionization begins at about 200 km altitude, the elevation angles for earth to satellite lines will always be at least 15 degrees.

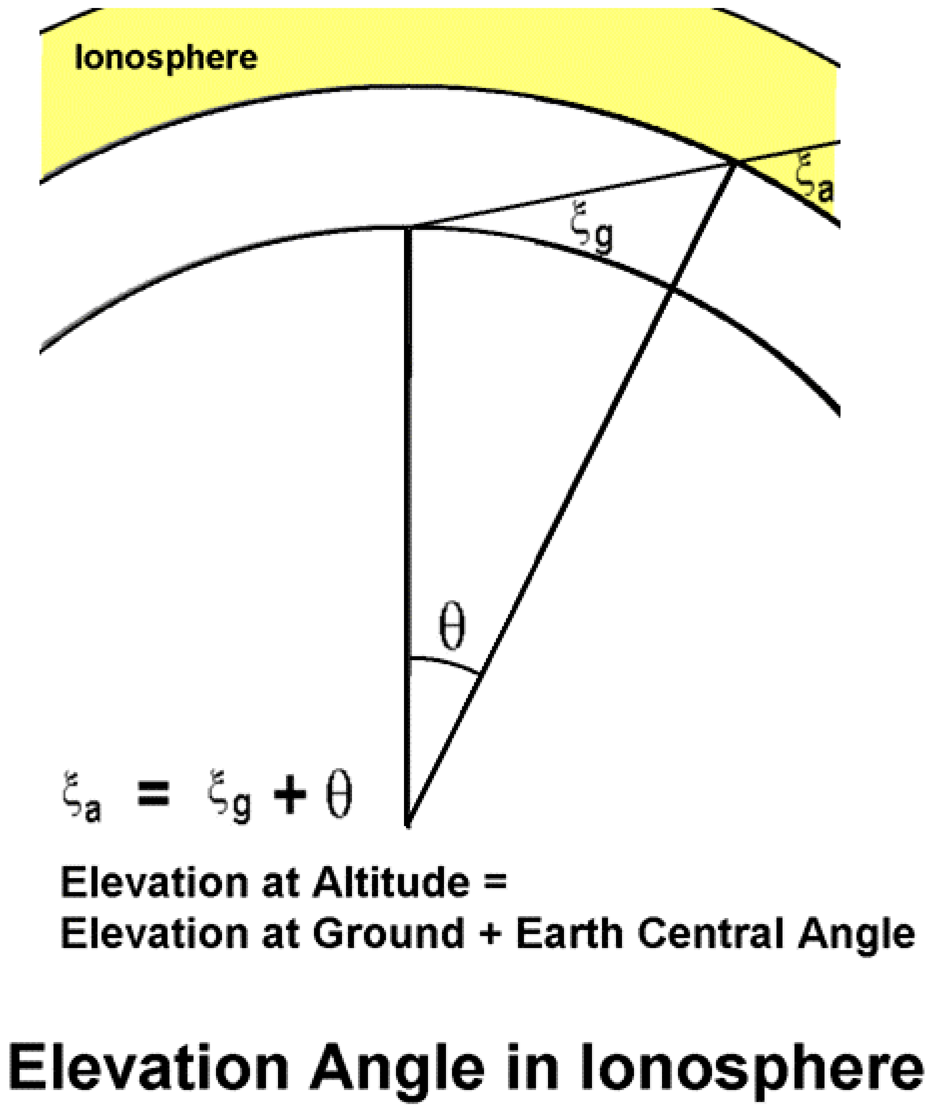


Figure 4

The change in the elevation angle due to the geometry change is the earth central angle between the take off point and the observation point. Any true refraction or bending will be in addition to this purely geometric effect.

The bending can be considered to take place at boundaries and the usual form of Snell's law applied. In the usual formulation, the angle of incidence is used. This is the complement of the elevation angle. So the elementary form of Snell's law is

$$n_1 \cos \xi_1 = n_2 \cos \xi_2 \ .$$

Here the n's are the refractive index within the given layers and the ξ's are the elevation angles.

This could be used at a sequence of layers. For the earth, it is best to have the layers be curved with the earth. In effect they become shells. Snell's law can be transformed for this case into the equation

$$nr \cos \xi = \text{constant},$$

where r is the radius from the center of the earth at each point.

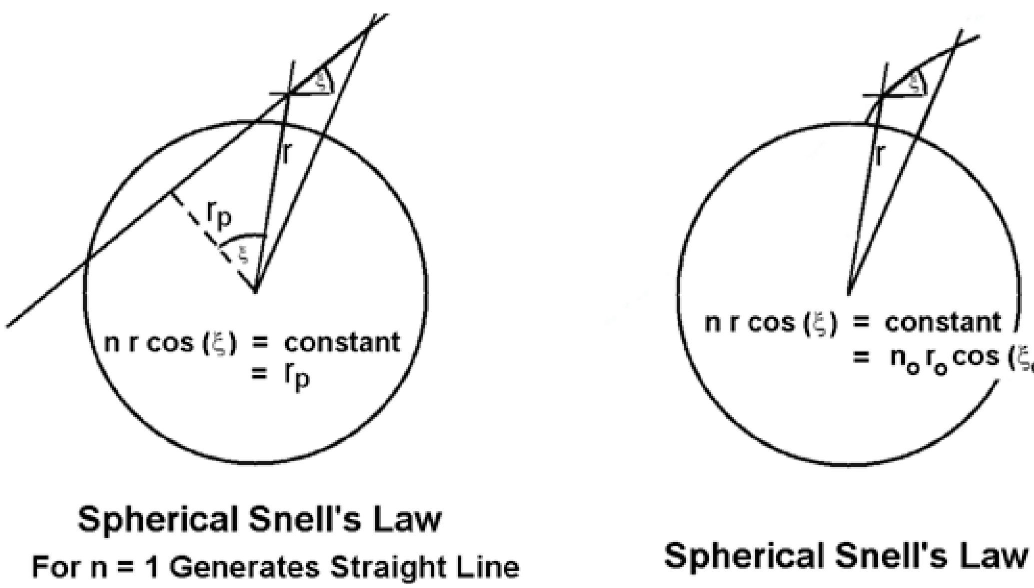


Figure 5

The left hand panel in Figure 5 shows the case of a fixed refractive index of unity. In this case the constant is easily found to be the shortest distance from the line to the center of the earth. In the more generic case, it is the value of the expression at the ground.

This leads to a relationship between the true bending and the horizontal offset of a ray caused by the ionosphere. When an earth to spacecraft ray is considered, the refractive index can be considered to be one at both end points. (The complications due to the atmospheric refractive index not being one are ignored here. When the bending effects are small, ionospheric and atmospheric bending may be computed independently of each other.) The direction of the ray will be the same on both sides for a plane slab. For the curved earth, the offset changes the local elevation angles and hence represents bending.

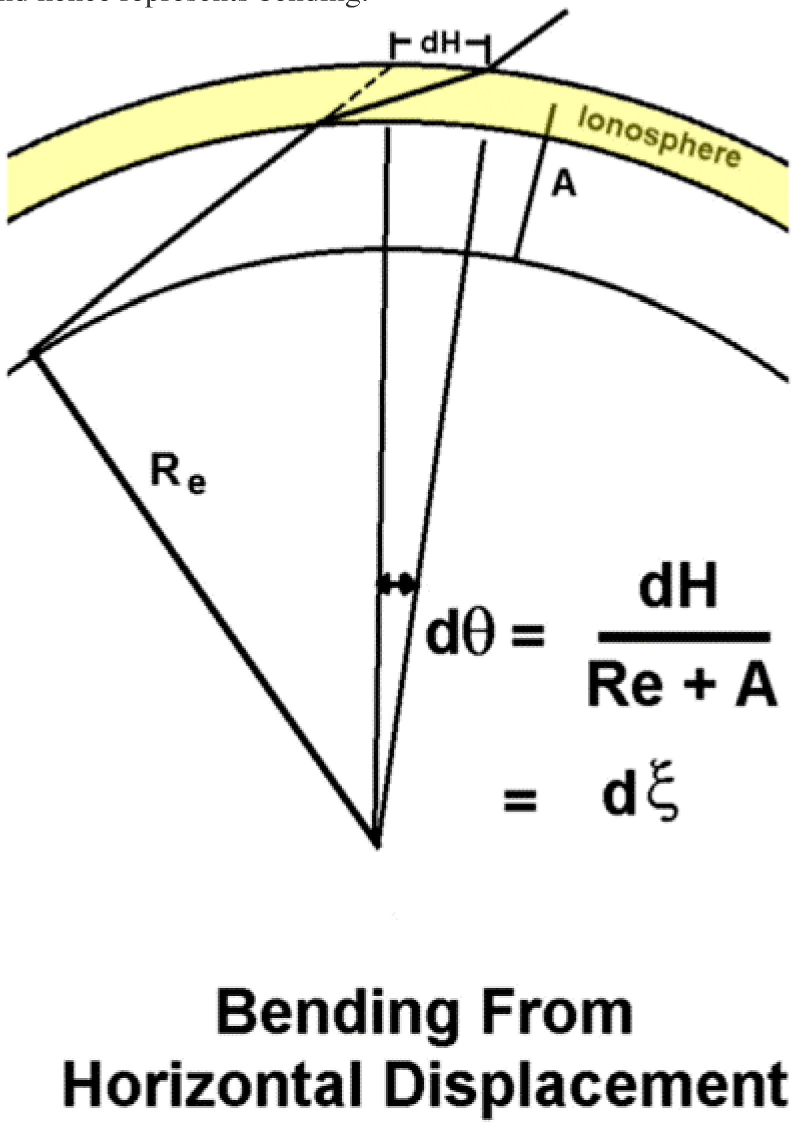


Figure 6

A ray going through the ionosphere is shown schematically in Figure 6. Here the ionosphere is modeled as a single shell with constant electron density. This simple model can give a rough estimate of effects. With a careful choice of parameters, it can give a conservative over estimate of any bending effects.

If the shell were a plane slab without bending, the ray above the ionosphere would be parallel to the ray below, but offset. In the case of the spherical shell there will be some net bending because the local vertical will not be the same at the true exit point that it would have been in the absence of the ionosphere.

The bending will be equal to the extra angle seen at the center of the earth caused by this offset. This is given by the equation in the figure.

V. Simulation

The simple shell ionosphere was programmed and a few simulations run. The parameters were chosen to represent maximum solar cycle maximum effects, a smaller level and a solar minimum case. Three ionospheric cases were run. In order to obtain an upper bound for bending, the ground elevation angle was assumed to be zero – that is the ray began horizontally.

The free parameters in the model were the altitude of the bottom of the layer, the layer top altitude, and the plasma frequency within the layer. In order to estimate the realism of the model, the vertical electron columnar content, N_v, was computed for the three cases.

$$N_v = \int N ds \, .$$

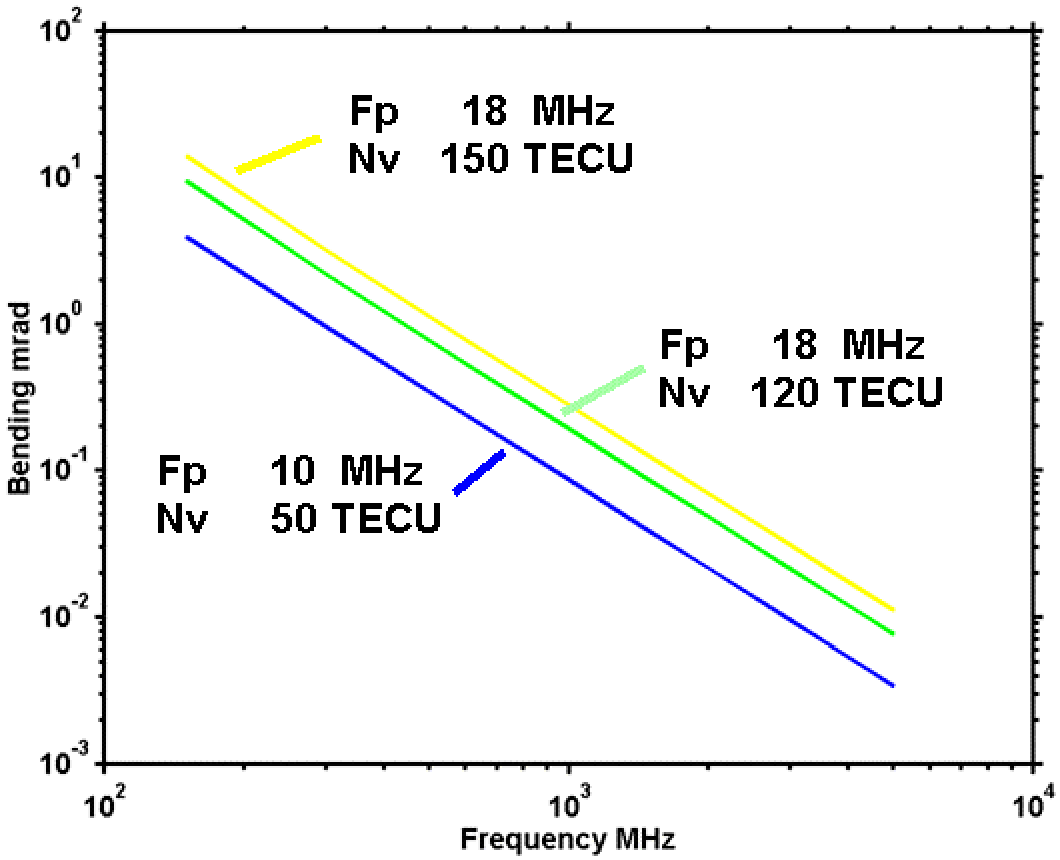
The integral is taken vertically through the ionosphere. This value is often reported in Total Electron Content Units (TECU) which are 10^{16} electrons/m^2 . Peak values of N_v in the tropics at solar maximum are about 150 TECU. During solar minimum the daytime maximums are about 20 to 50 TECU. The values for the models were 150, 120, and 50 TECU respectively.

One can analytically compute the approximate value of the ionospheric bending for a ground elevation angle of 0 degrees. It turns out to be a constant times $(f_p/f)^2$, where the constant is a function of the shell parameters. This approximation uses a curved earth to find the entry elevation angle, a horizontal slab for the ionosphere and curved earth above it. The numeric computation was done to remove some of these assumptions. However the functional dependence of the bending on frequency was verified.

The results of the numeric computation, using a full curved earth shell model are shown in Figure 7. All simulations were run for a ground elevation angle of 0 degrees. The parameters for the shell in the three cases are also shown on the figure. Frequencies between 150 MHz and 5 GHz were used. The plot is on a log-log scale to accommodate the large range of values.

The expected form of the line occurs; the bending is proportional to $(f_p/f)^2$. There is a slight bow to the curves, but this is not important given that this is a rough, order of magnitude estimate.

At 150 MHz, the lowest frequency modeled, the total refraction was 14 mrad, 9.5 mrad, and 4 mrad for the three cases respectively. This should be a good estimate for the maximum size of the bending in low, medium and high solar activity. There will be some day to day variations, but these can be expected to be less than 25 percent of the total. The values all scale as f^{-2} where f is the operating frequency.



Bending for Shell Ionosphere

Ground Elevation Angle 0 Degrees

Parameters for 3 Cases

Fp MHz	18	18	10
A-Bottom Km	200	250	200
A-Top Km	600	550	600
Nv TECU	150	120	50

Figure 7