Note on Heights Types used in Geodesy

James R. Clynch
February 2006

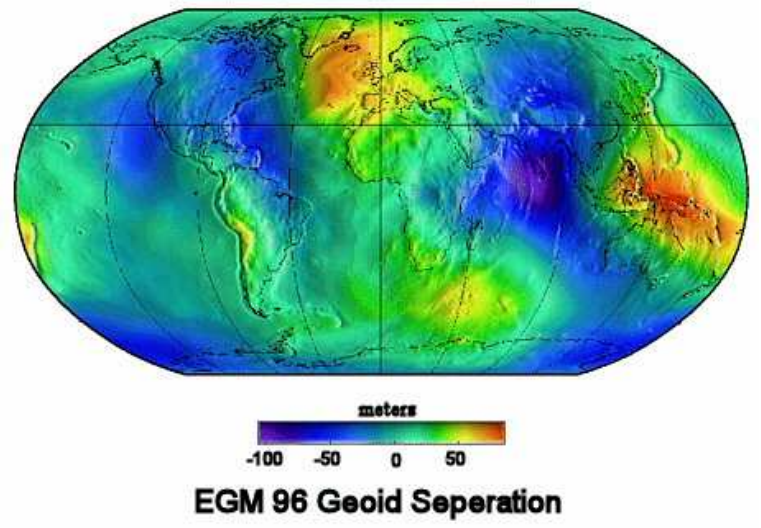
Rather than define the radial distance from the center of the earth for a point, it is more convenient to define the height. The question is what is the reference surface. For the spherical earth model, the answer is simple, the sphere with the radius of the mean earth is the reference.

For the ellipsoidal model of the earth, the ellipsoid can be used as the reference surface. This is done and the resulting height is called ellipsoidal height or geodetic height. But this is not the height found on maps.

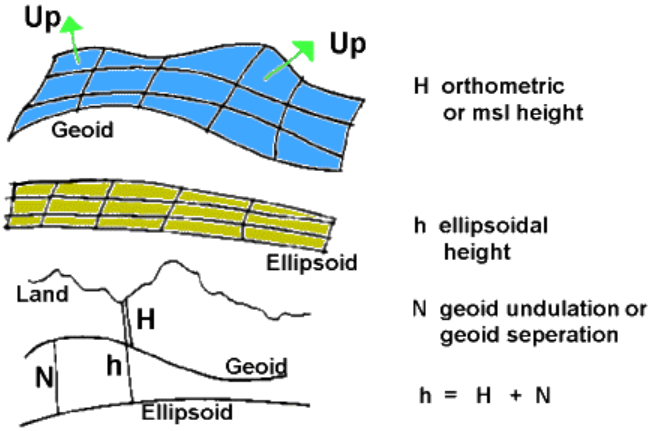
For practical purposes the height used on maps is mean sea level (MSL) heights. It has the official name of orthometric height. It is measured by surveying from point to point inland from points near the sea where mean sea level is determined by tide gauges. This is the height on maps.

One might assume that geodetic (ellipsoidal) height and MSL height would just be different by some constant that is the error in the tide gauge calibration. But this is not the case. The sea is not "flat", that is it is not an ellipsoid. The true gravity field of the earth is lumpy.

At each survey station the local vertical is found using via a level. This causes the survey to follow a surface of constant gravity potential, not the smooth ellipsoid. This surface is called the geoid. It is the extension of sea level. It can be above or below the ellipsoid. For a well chosen ellipsoid, the difference is less than 100 m everywhere on the earth.



Thus there are two heights corresponding to the two reference surfaces. The operationally defined heights are msl or orthometric heights. They use the geoid as the reference. These will be called H here. The heights from the ellipsoid, geodetic or ellipsoidal heights, will be called h. The difference between then, the height of the geoid measured from the ellipsoid is called the undulation of the geoid, the geoid separation, or just the undulation. It is denoted by N. N must be measured, it is determined by the lumps, on and within the real earth. This is very hard to do at high accuracy.



Geoid - Ellisoid Diagram
Two and Three Dimensions

Height Types

Orthometric Mean Sea Level (msl)	H	Measured from Geoid Found on Maps
Geodetic Ellipsoidal	h	Measured from Ellipsoid Used in equations
Undulation Undulation of Geoid	N	Distance from Ellipsoid to Geoid Can be positive or negative
		N = h - H h = H + N

If you determine a XYZ (Cartesian) coordinate of a point, you can easily find the geodetic height h. Satellite based positions work this way. GPS positions are inherently Cartesian locations and directly produce geodetic or ellipsoidal height. Any msl height produced is the result of using a geoid undulation model. The error in H from GPS can be on the order of a meter for absolute positions due to the geoid model used.

If you determine a height by classical surveying you get msl or orthometric heights. Good surveys are accurate at the few cm level. N can be measured in several different ways. Today the difference in N at near survey points has little error. But the error over the US might be 10's of cm. This is improving however.

Note on heights in meteorology:

There is another common height, used in meteorology, called geopotential height (gph). Each meter in gph is separated by the same gravitational potential. The gph meter increases as you go up in altitude due to the small decrease in the gravity field as you move away from the earth's center. The gph is often represented by the symbol "z".

Approximations adequate for meteorology are easy to compute. Using geopotential height simplifies several meteorology equations. Also in meteorology they sometimes refer to "geometric altitude" which is msl altitude or orthometric height.