Latitude and Longitudes in Geodesy

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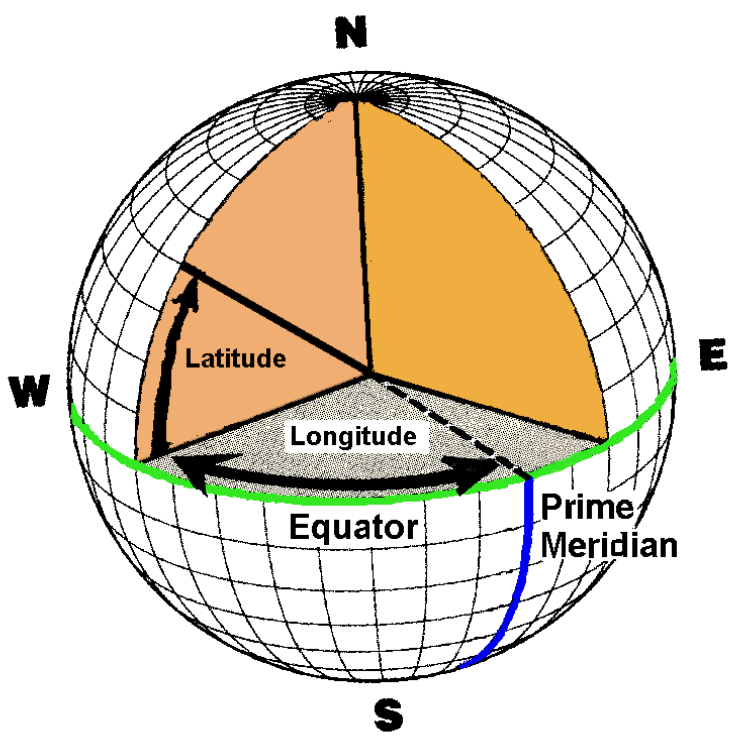
I. Latitude and Longitude on Spherical Earth

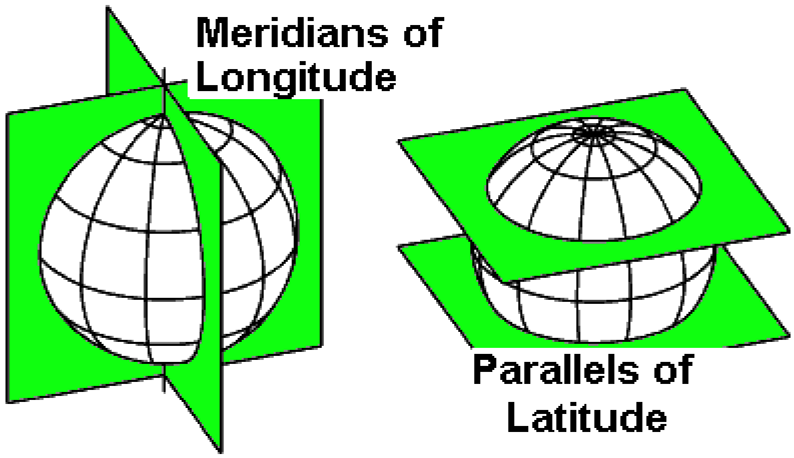
Latitude and longitude are the grid lines you see on globes. For a spherical earth these are angles seen from the center of the earth. The angle up from the equator is latitude. In the southern hemisphere is it negative in the convention used in geodesy. It has a range of –90 degrees to 90 degrees. The reference for latitude is set by the equator - effectively set by the spin axis of the earth.

The angle in the equatorial plane is the longitude. There is no natural reference for longitude. The zero line, called the prime meridian, is taken, by convention, as the line through Greenwich England. (This was set by treaty in 1878. Before that each major nation had its own zero of longitude.)



The longitude used in geodesy is positive going east from the prime meridian. The values go from 0 to 360 degrees. A value in the middle United States is therefore about 260 degrees east longitude. This is the same as -100 degrees east.

In order to make longitudes more convenient, often values in the western hemisphere are quoted in terms of angles west from the prime meridian. Thus the longitude of -100 E (E for East) is also 100 W (W for West). Similarly latitudes south of the equator are often given as "S" (for south) values to avoid negative numbers.



The lines of latitude and longitude form a pattern called the graticule. The lines of constant longitude are called meridians. These all meet at the poles. They are circles of the same radius which is the radius of the earth, R_e. They are examples of great circles. The lines of constant latitude are called parallels of latitude. They are circles who's radius dependents on the latitude. The radius is $\cos(\phi) R_e$. Parallels of latitude are examples of small circles.

II. Latitude and Longitude on Ellipsoidal Earth

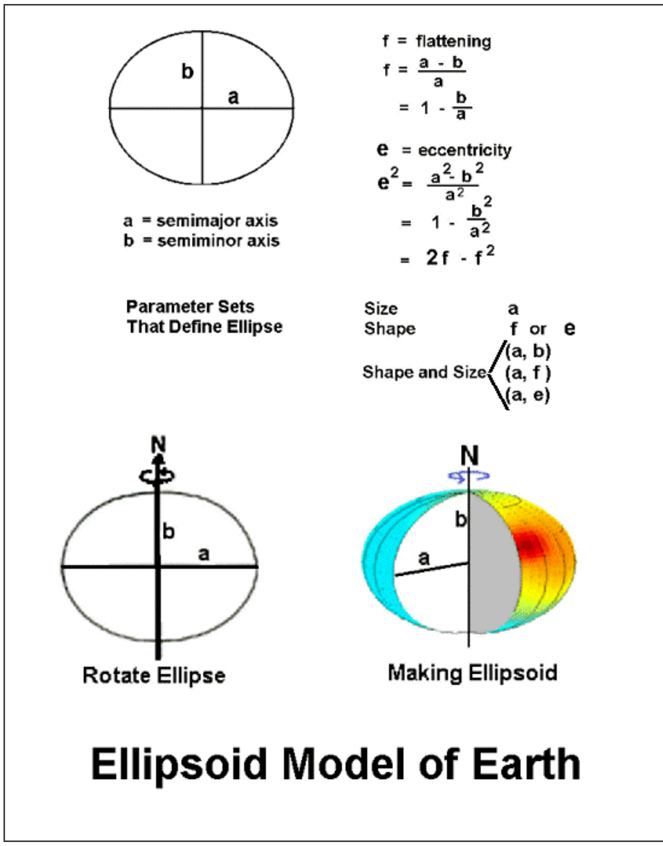
The earth is flattened by rotational effects. The cross-section of a meridian is no loner a circle, but an ellipse. The ellipse that best fits the earth is only slightly different from a circle. The flattening, defined in the figure below, is about 1/298.25 for the earth.

Latitude and longitude are defined to be "intuitively the same as for a spherical earth". This loose definition has been made precise in geodesy. The longitude is the exactly the same as for a spherical earth. The way latitude is handled was defined by the French in the 17th century after Newton deduced that the world had an elliptical cross-section.

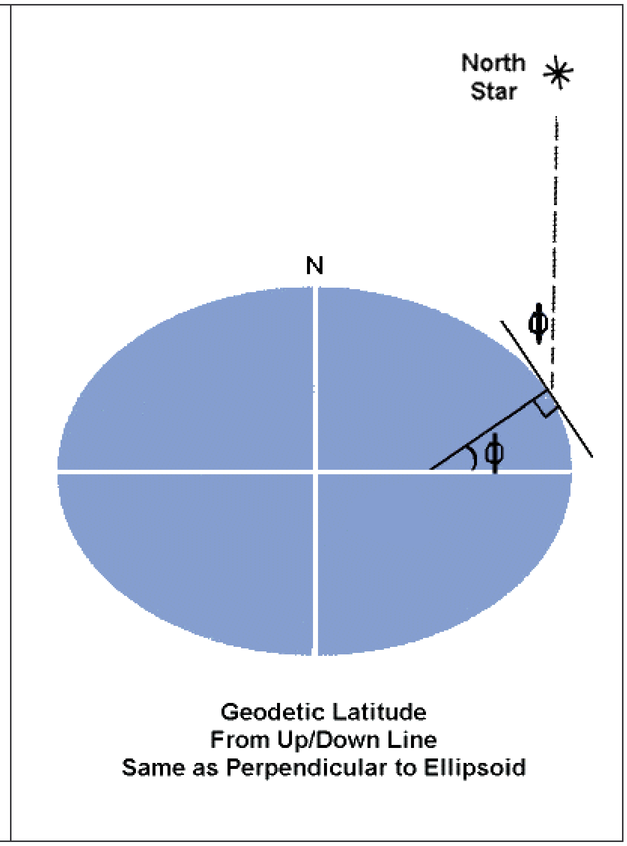
Before satellites latitude was measured by observing the stars. In particular observing the angle between the horizon and stars. The horizon was taken to be perpendicular to the vertical measured by a plumb bob or spirit level. The "vertical line" of the plumb bob was thought to be perpendicular to the sphere that formed the earth. The extension to an ellipsoidal earth is to use the line perpendicular to the ellipsoid to define the vertical. This is essentially the same as the plumb bob.[1]

[1] There is a very small difference between the vertical defined by a plumb bob, and the perpendicular to the ellipsoid. This is only a few arcseconds. Latitude defined from a plumb bob or spirit level is technically called Astrodetic latitude.

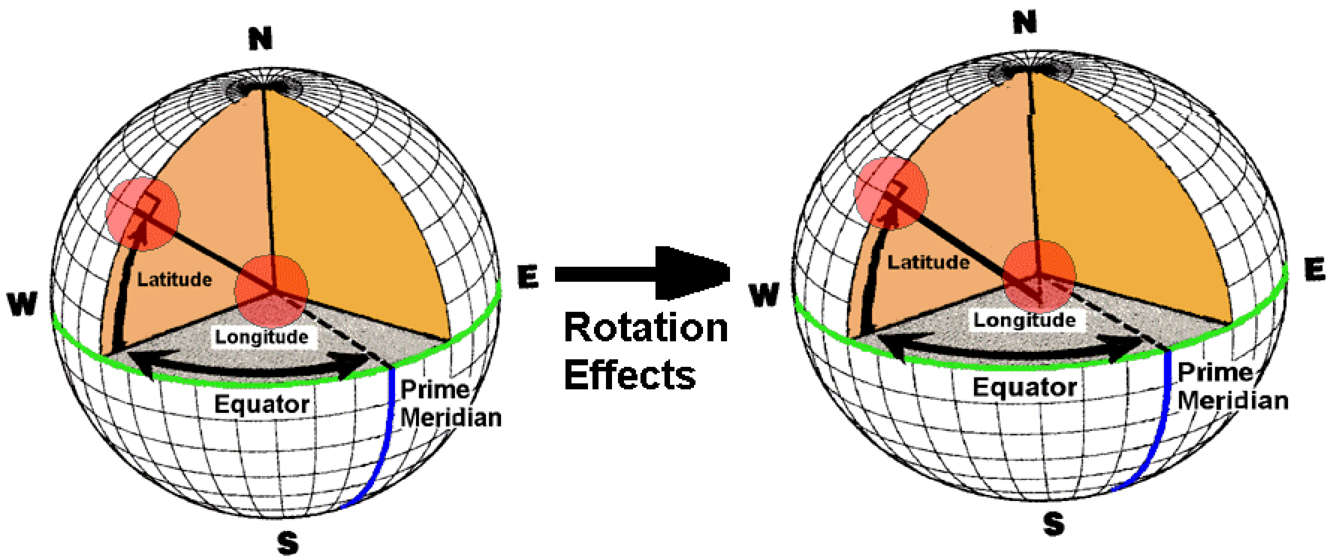
The figures below show the key effects of rotation on the earth and coordinates. The latitude is defined in both the spherical and ellipsoidal cases from the line perpendicular to the world model. In the case of the spherical earth, this line hits the origin of the sphere - the center of the earth. For the ellipsoidal model the up-down line does not hit the center of the earth. It does hit the polar axis though.



Ellipsoid Model of Earth



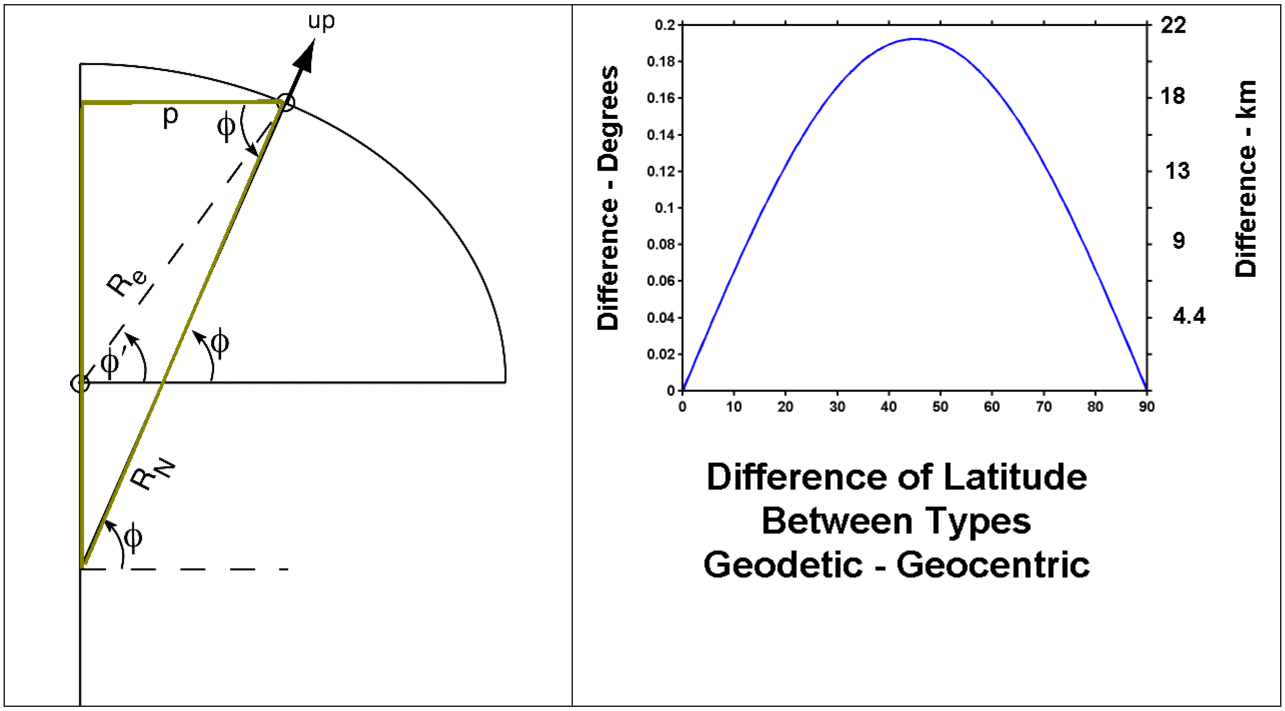
Geodetic Latitude
From Up/Down Line
Same as Perpendicular to Ellipsoid



Rotation Effects Change Latitude Defination

The length of the line to the center of the earth for a spherical model is the radius of the sphere. For the ellipsoidal model the length from the surface to the polar axis is one of three radii needed to work with angles and distance on the earth. (It is called the radius of curvature in the prime vertical, and denoted R_N here. See the note on radii of the earth for details.)

There are not two types of latitude that can easily be defined. The angle that the line makes from the center of the earth is called the geocentric latitude. Geocentric latitude is usually denoted as ϕ', or ϕ_c. It is commonly used in satellite work. It does not strike the surface of the ellipsoid at a right angle. The line perpendicular to the ellipsoid makes an angle with the equatorial plane that is called the geodetic latitude. (“Geodetic" in geodesy usually implies something taken with respect to the ellipsoid.) The latitude on maps is geodetic latitude. It is usually denoted as ϕ, or ϕ_g



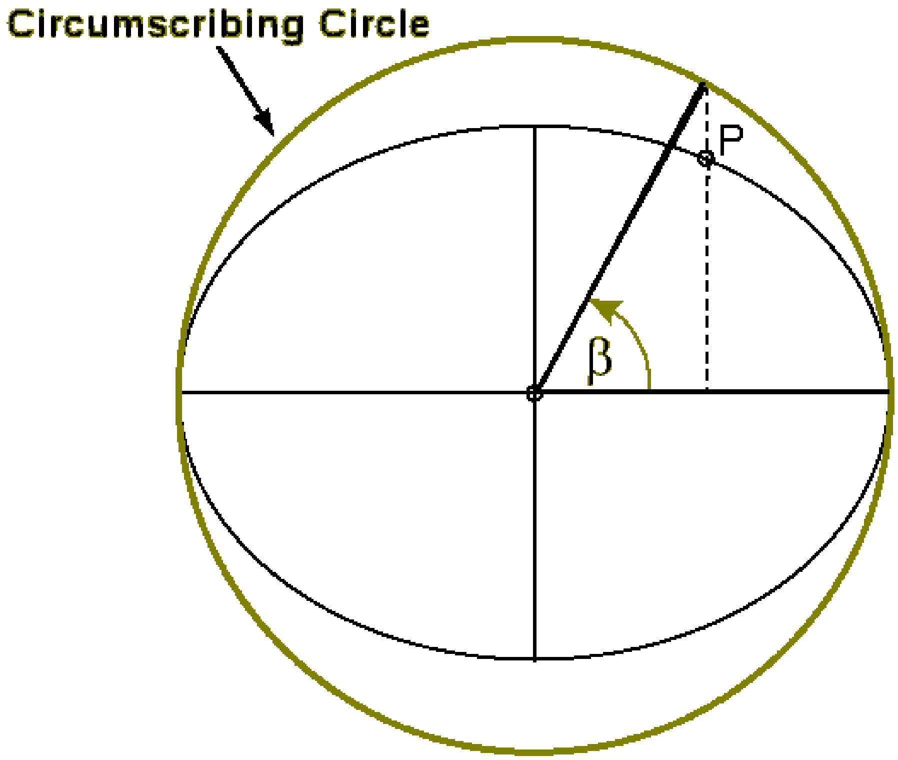
Above on the left is a diagram showing the geodetic (ϕ) and geocentric (ϕ') latitudes. The physical radius of the earth, R_e, and the radius of curvature in the prime vertical, R_N, are also shown. The highlighted triangle makes it clear that the radius of the parallel of latitude circle, called p, is just $R_N \cos(\phi)$, where the geodetic latitude is used.

From the figure it is also clear that the geodetic latitude is always greater or equal in magnitude to the geocentric. The two are equal at the poles and equator. The difference for the real world, with a flattening of about 1/300, is a small angle. However this can be a significant distance on the earth surface. The difference in degrees and kilometers on the earth surface are shown on the right above.

III. Other Latitudes

There are many other "latitudes" that show up in geodesy books. The two that are most important are Astrodetic Latitude and Reduce Latitude. Most other forms are specialized. Several are used only in one or two map projections.

Astrodetic latitude is obtained from direct observations of stars using the local gravity field to define up (using a plumb bob or spirit level). There is also an astrodetic longitude. The difference between astrodetic and geodetic coordinates is small, usually a few arcseconds.



Reduced Latitude

Reduced latitude, usually denoted by β in geodesy, is an angle from the center of the earth to a circle that has the semi-major axis length. This is called the circumscribing circle. The angle is to the point on the circumscribing circle that is along a line parallel to the Z axis and through the point. This angle is used in theoretical gravity studies, particularly in the pre-computer age. This angle is important in orbit work. There it is called the Eccentric Anomaly and denoted by "E".