Earth Models and Maps

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I. Earth Models

A map is just a model of the world, or a small part of it. This is true if the model is a globe of the entire world, a paper chart of a harbor or a digital database of streets in San Francisco. A model of the earth is needed to convert measurements made on the curved earth to maps or databases. Each model has advantages and disadvantages. Each is usually in error at some level of accuracy. Some of these error are due to the nature of the model, not the measurements used to make the model.

Three are three common models of the earth,

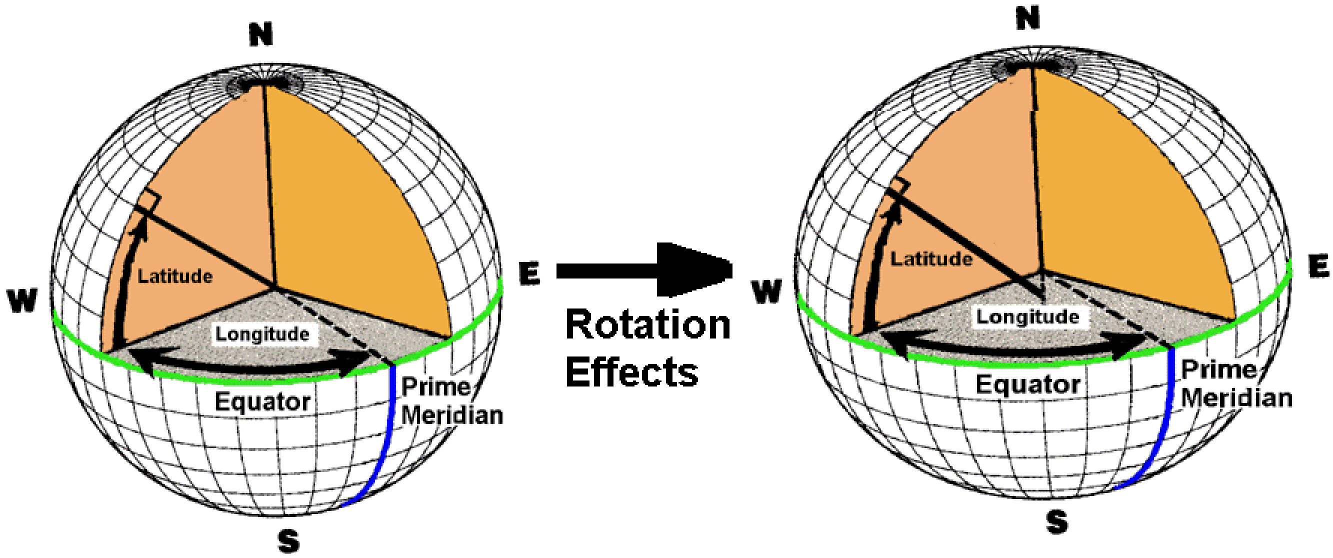
the **spherical (or globe) model**,
the **ellipsoidal model**, and
the **real earth model**.

The **spherical model** is the form encountered in elementary discussions. It is quite good for some approximations. The world is approximately a sphere. The sphere is the shape that minimizes the potential energy of the gravitational attraction of all the little mass elements for each other. The direction of gravity is toward the center of the earth.

This is how we define down. It is the direction that a string takes when a weight is at one end - that is a plumb bob or plumb line. A spirit level will define the horizontal which is perpendicular to up-down.

The **ellipsoidal model** is a better representation of the earth because the earth rotates. This generates other forces on the mass elements and distorts the shape. The minimum energy form is now an ellipse rotated about the polar axis. This is called an **ellipsoid**. (It is called a spheroid in some books.) The equatorial radius is longer than the polar axis by about 23 km. The direction of gravity does not point to the center of the earth. We still call the direction of a plumb bob down and use it to define coordinates.

In the ellipsoidal model the direction of down can be shown to always be perpendicular to the ellipsoid. Thus the ellipsoid must be a surface of constant gravity potential. (Notice that in geodesy, “gravity” means the observed force. This is the sum of the Newtonian gravity and rotational effects.) Fluids don't flow along the ellipsoid due to gravity. Gravity only pulls perpendicular to the ellipsoid. In the real world this will be slightly incorrect.



Rotation Effects Change Shape of Earth and Latitude Definition

The **real world** is not homogeneous. There are mass variations such as oceans and mountains. There are also inhomogeneities under the surface. These cause not only the mountains but also variations in the gravity field. Thus the measured down direction is changed.

The differences between the ellipsoidal down and the true down is very small. However they change the fundamental surface we use for height measurements. For practical reasons heights are measured from a bumpy surface everywhere perpendicular to the real down. There are many of these constant potential surfaces, called **level surfaces**[1]. We call the level surface that represents mean sea level in the open ocean the **geoid**. This is a surface of constant gravity potential - a level surface. Down is always perpendicular to the local level surface.

II. World Models and Coordinates

We use coordinates on maps to label points and make measurements. The most common set is latitude, longitude, and height. In the earliest maps, the spherical model was used. The ellipsoidal shape of the earth was discovered by Newton in the 1600's. From then on mapmakers used an ellipsoid as a model of the earth to analyze

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[1] **Level surfaces** are surfaces of constant gravity potential. They are the three dimensional analog of contour plots in two dimensions. They are the locations where a function has some specific value. This function is the potential energy per unit mass of something rotating with the earth. The sea is one level surface. (If we ignore some small effects due to ocean currents.)

measurements. This placed latitude and longitude on the ellipsoidal model. However the heights were on the real-world model.

The table below gives a summary of the key properties of these three model. Notice that he spherical form is not used for the coordinates we see on maps. We see that the latitude and longitude are from the Ellipsoidal Model. The height is from the Real World Model.

MODEL	Surface	Latitude Longitude	Height
Spherical/Globe Used in elementary descriptions	Sphere	Geocentric	spherical
Ellipsoid/Ellipsoidal Used in mapmaking	Ellipse of Revolution	Geodetic **Used on maps**	ellipsoidal Produced by Satellite Systems (GPS etc.)
Real World	Geoid A Level Surface	Astrodetic or Astronomic	Orthometric (Mean Sea Level) **Used on maps**

So spherical model coordinates are only used for general discussions and education. Latitude and longitude on maps are from the ellipsoid model. Heights are on the real world model. The way surveyors make maps causes this.

The earth falls off significantly with distance. Surveyors must make adjustments in analysis of measurements of angles and distances because the world is not flat. They could use a spherical model, but that generates unacceptable errors. So an ellipsoid is used to make the adjustments. This form of latitude and longitude are called "geodetic". In general when the term geodetic is used, it means that an ellipsoid was used to model the earth.

Heights are measured from some reference mark that is tied to mean sea level. This is often a bench mark near a tide gauge. Surveying outward from these primary height marks, the local gravity field is used to define down and the horizontal. This causes heights to follow the geoid.

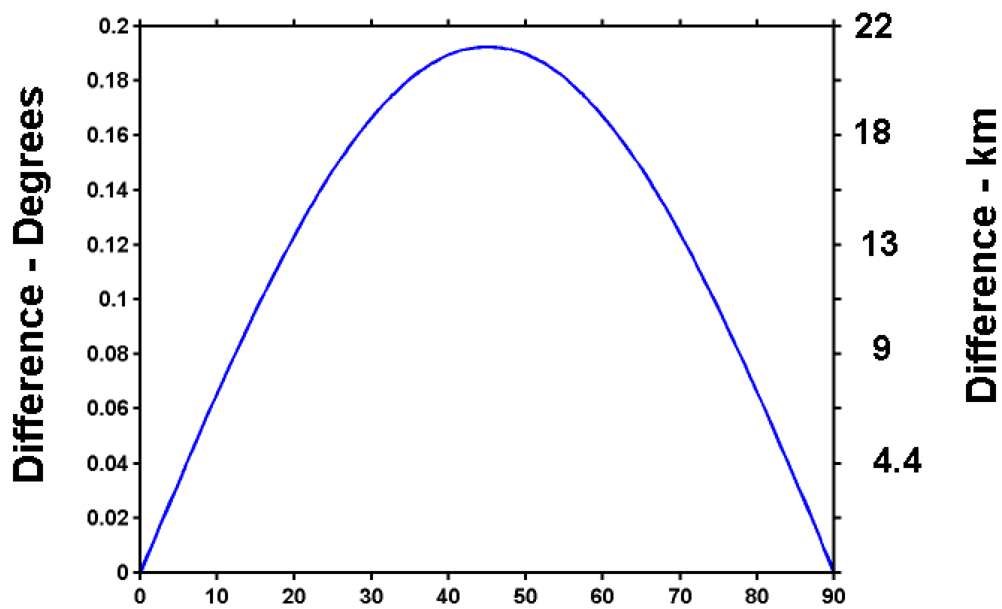
III. Difference Between Models

The difference between the spherical and ellipsoidal models is a function of the location and, for lengths, the distance. In general the errors are relative, being about the flattening of the real earth times the distance. The flattening of the earth is about 1/300. So if you need an answer more accurate that 1/300 you probably need to use the ellipsoid model. This is about 3 m per kilometer or 20 ft per mile.

The difference between the ellipsoidal and real world model is much smaller. Where the differences between the spherical and ellipsoidal latitudes can be about 10 arc minutes, the difference between the real world and ellipsoidal model are on usually 10 arc seconds or less.

A. Latitude and Longitude

The errors in using the spherical model can be significant. One way to express the difference between the spherical and ellipsoidal model is to look at the difference in the latitudes. The **geocentric latitude** is measured from the line to the center of the earth. This is not used on maps. It is, however, used in satellite work. The **geodetic latitude** is always greater or equal to the geocentric latitude. The two are equal at the poles and equator and have greatest difference at 45 deg north and south.



Difference of Latitude Between Types Geodetic - Geocentric

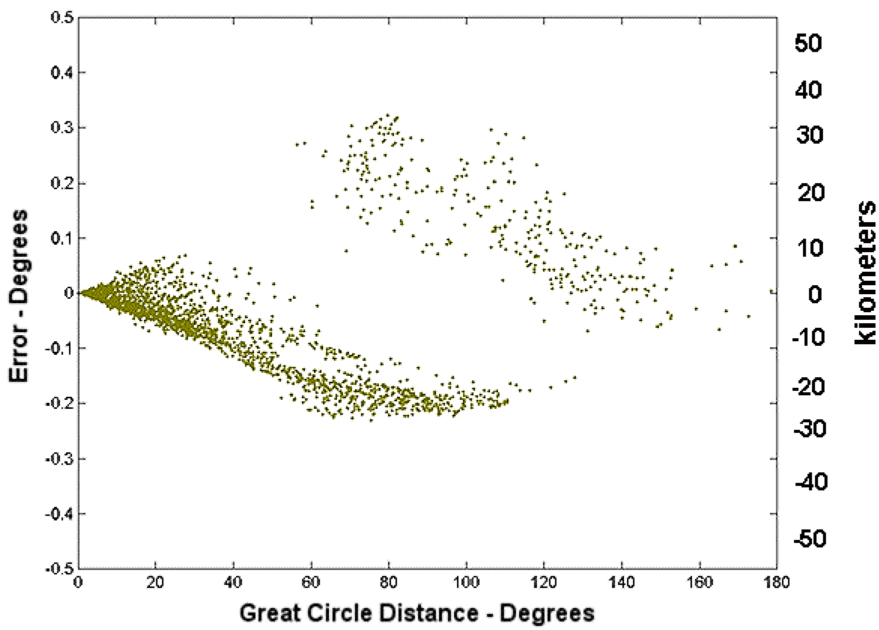
On this figure the angular units are shown on the left. This is converted to distance on the ground on the right axis. It is clear that you cannot navigate or do any quantitative work using geocentric latitude.

While latitude is complicated by the ellipsoidal shape of the earth, longitude is not. Geodetic and geocentric longitude are identical.

Another way to illustrate the magnitude of the errors in the spherical model is to compute distances To illustrate this 40 major cities were chosen. Twenty were in the US and the other 20 were world capitals. The distance between each pair was computed twice, once with a spherical earth and once with an ellipsoid model. The difference are shown below.

For the spherical computation, the radius used was the geometric average of the true radii at each end point. This gives a much better value than using one radius for all locations. Therefore this is better than a true spherical model.

Notice that for short distances, the error is small. Of course small is a relative term and depends on the application. For road driving between San Francisco and Salt Lake City the error of a couple of km is not important. For entering a harbor a 100 m may be huge.



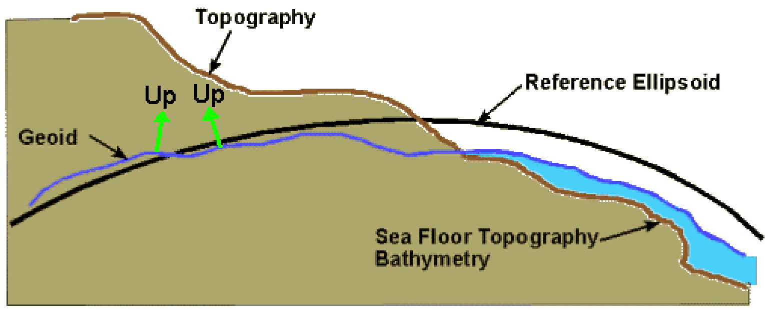
Distance Error
Spherical Earth Assumption

All Combinations of 20 US Cities and 20 World Capitals

B. Heights

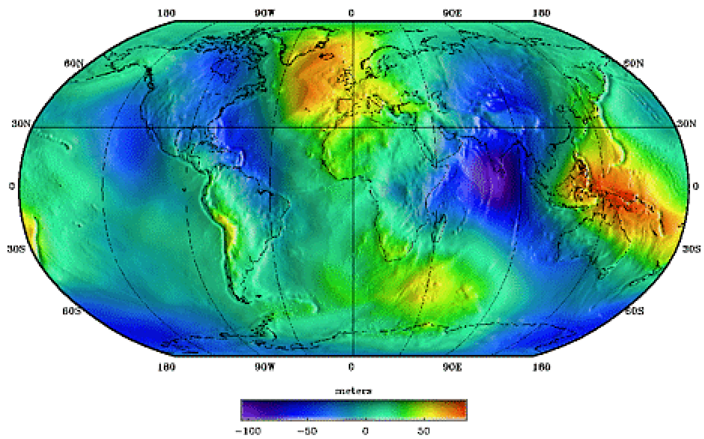
Surveyors cannot really find the ellipsoid. But they can find the sea surface. Surveying inland from the sea gives height with respect to sea level. In the process of these surveys the equipment is always set level - that is perpendicular to the local down. This causes the heights to be measured with respect to the geoid.

There are three surfaces that are used in measuring heights. For maps we use the **geoid**. We commonly call this **mean sea level heights (msl).** These are formally called **orthometric heights**. For satellite work, the ellipsoid is used. Heights measured from the ellipsoid are called **ellipsoidal heights**. For aircraft the height **above ground level** (**agl**) is sometimes used. For that purpose the topography is the reference surface.



Three Surfaces of Geodesy

The differences between heights measured with the spherical and ellipsoidal model are on the order of 20 km at the poles. The differences between ellipsoidal and orthometric (map) heights are much smaller. They vary over a range of -100 m to +100 m. The average over the world is zero. (This is one way to define the ellipsoid) The distance from the ellipsoid to the geoid is called the undulation of the vertical or just the undulation. A map of the undulation made in the 1990's is shown below.



Undulations
Model Geoid-50