

Notes on Projections

Part II - Common Projections

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I. Common Projections

There are several areas where maps are commonly used and a few projections dominate these fields. An extensive list is given at the end of the Introduction Chapter in Snyder. Here just a few will be given.

Whole World

Mercator	Most common world projection. Cylindrical
Robinson	Less distortion than Mercator. Pseudocylindrical
Goode	Interrupted map. Common for thematic maps.

Navigation Charts

UTM	Common for ocean charts. Part of military map system.
UPS	For polar regions. Part of military map system.
Lambert	Lambert Conformal Conic standard in Air Navigation Charts

Topographic Maps

Polyconic	US Geological Survey Standard. UTM coordinates on margins.
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Surveying / Land Use / General

Adlers Equal Area (Conic)	
Transverse Mercator	For areas mainly North-South
Lambert	For areas mainly East-West

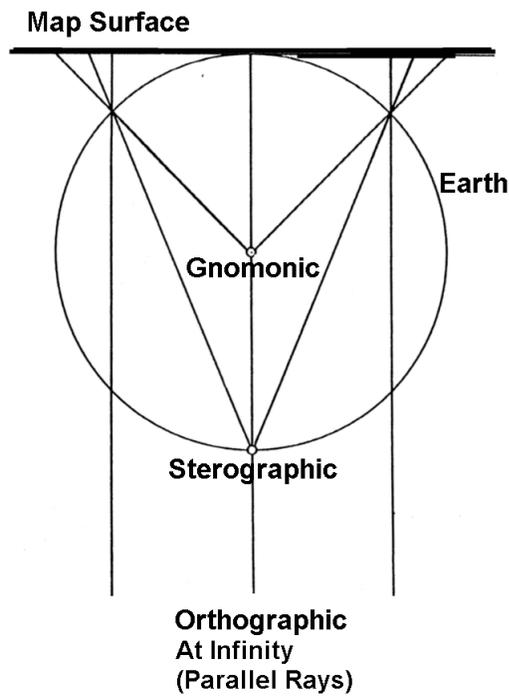
A discussion of these and a few others follows. For an extensive list see Snyder.

II. Azimuthal or Planar Projections

If you project a globe on a plane that is tangent or symmetric about the polar axis the azimuths to the center point are all true. This leads to the name Azimuthal for projections on a plane. There are 4 common projections that use a plane as the projection surface. Three are perspective. The fourth is the simple polar plot that has the official name equidistant azimuthal projection.

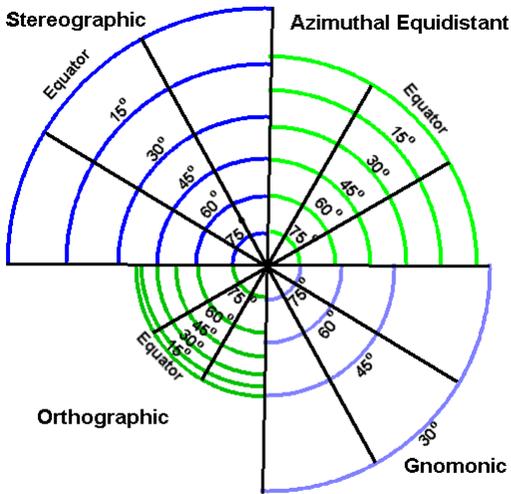
The three perspective azimuthal projections are shown below. They differ in the location of the perspective or projection point. These are:

Azimuthal Projection	Perspective Point
Stereographic	Opposite point from tangent
Gnomonic	Center of Earth
Orthographic	Infinity
Equidistant	None

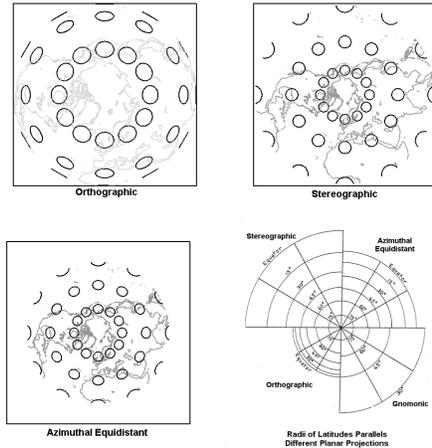


Azimuthal or Planar Projection Points

The major difference of these projections evident on a map is the manner that the parallels of latitude are spaced on the map surface. This is shown in the figure below on the left. The equator appears on all but the Gnomonic. (The equator is at infinity on a Gnomonic projection.) The spacing is even only for the equidistant azimuthal (simple polar plot). Tissot's Indicatrix is shown for the same for projections on the right. Only the stereographic has circles everywhere and is therefore conformal. On all the others angles are not true on the map.



Radii of Latitude Parallels For Common Azimuthal Projections



Tissot's Indicatrix Common Azimuthal Projections

A. Stereographic Projection

This is the most common azimuthal projection used on standard maps. It is conformal. The **Universal Polar Stereographic (UPS)** is used in the polar regions for military maps. Like the UTM, the projection surface is depressed into the earth to control the maximum scale error.

B. Orthographic

This projection has the "light rays" coming in parallel. This is often expressed as having the projection point at infinity. It is often used for maps of a half earth from pole to pole because it emphasizes the roundness of the earth. In the example below, the tangent point is on the equator. (You could also say that this is a transverse projection.)



Orthographic Projection
Centered at 0 N 45 E

One of the valuable uses of an Orthographic Projections is for maps to compare to aerial photographs. Aerial photographs taken with long focal length lenses are almost the "view from infinity". In

most cases when you get an aerial photograph you do not receive the raw image. Instead you get an **orthorectified** or **ortho photo**. In this case the photograph is the same as an orthographic map. There is more to the orthorectification process.

C. Gnomonic

This projection is not conformal, not equal area, and badly distorts shapes. But it has one advantage for the navigator, straight lines on a Gnomonic Projections are Great Circles. This means that the shortest path to sail between two points can be found by just drawing a straight line on a Gnomonic map. This occurs because a great circle is the intersection of a plane that goes through the center of the earth and the sphere. The intersection of this plane and the Gnomonic projection, which is projected from the center of the earth will then be a straight line that passes through the same points on the map and the sphere.

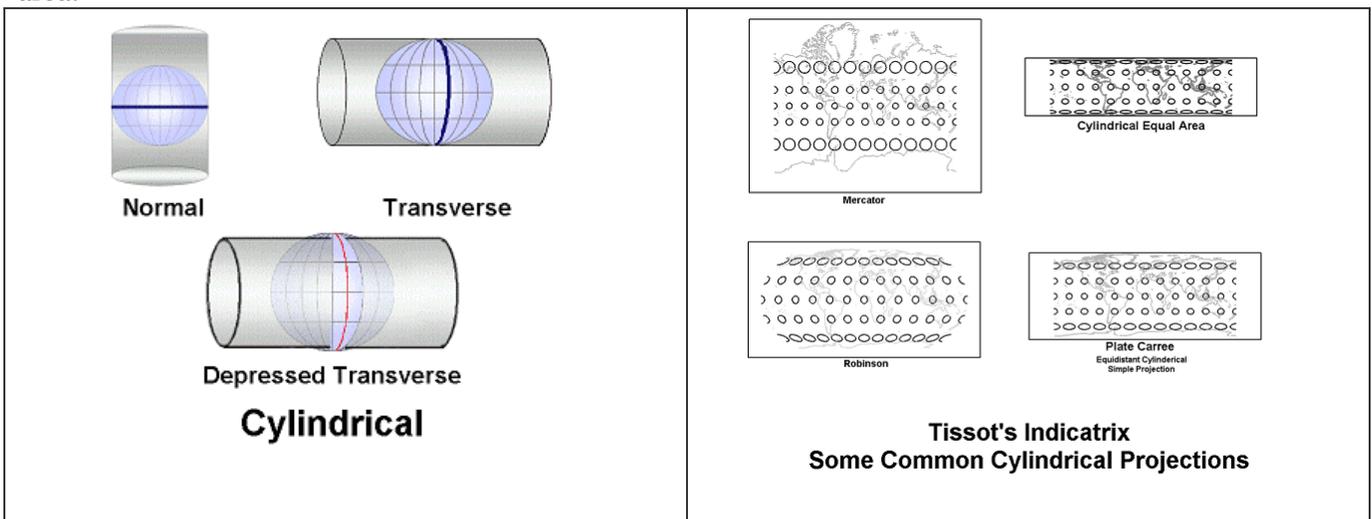
But the azimuth will be changing constantly along this line. Navigators used to plot a course on a Gnomonic, then take the latitude and longitude at 100 mile increments off this map and re-plot the points on a Mercator projection. Connecting the points produced a series of constant azimuth segments that closely approximated the shortest distance path. (Lindbergh used this technique on his transatlantic flight.)

D. Equidistant Azimuthal

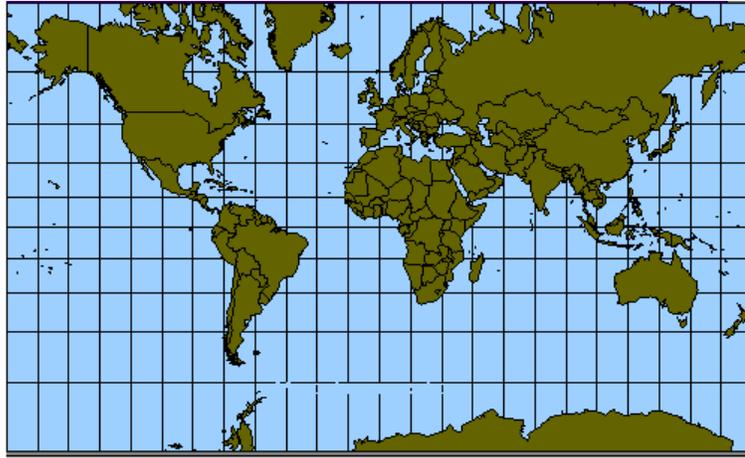
This is the common polar projection. It does a reasonable job for polar regions and is found in several atlases for the polar plots. It is neither conformal or equal area.

III. Cylindrical Projections

The three common geometries for cylindrical projections are shown on the left below. On the right are Tissot's Indicatrix diagrams of 4 common cylindrical projections. The Mercator is the only one that has circles everywhere. It is conformal. The others are not conformal. Only the equal area cylindrical is equal area.

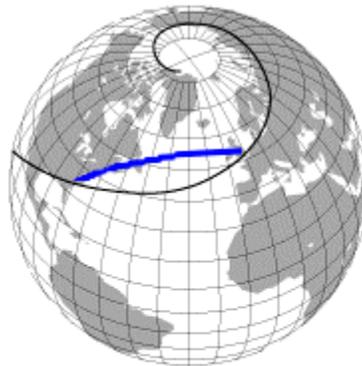


A. Mercator



Mercator Projection

The Mercator projection is a conformal, cylindrical projection. It was invented to produce maps of large areas that were conformal. The idea is that drawing a straight line on the map between two points produces the correct sailing azimuth. If you measure the azimuth on the map and then sail this azimuth constantly, you will reach the destination. A line of constant azimuth is called a **rhumb line**. This was of great practical importance to sailors.



**Rhumb Line (Loxodrome)
And Great Circle**

The rhumb line and the shortest path between Washington and London are shown on the map above. Clearly neither is a straight line and they are different. The rhumb line has been extended in both directions. This type of curve has the name loxodrome. It spirals to the pole.

The Mercator projection is produced by enclosing the earth in a cylinder and drawing lines from the polar axis out. In simple geography books these lines are shown coming from the center of the earth. That is the case for points on the equator. Points poleward are drawn from points on the axis up or down from the center. The Mercator is not the shadow of the earth on a cylinder from a light bulb at the center of the earth as often shown in elementary books.

Mercator had a geometric method of determining how far away from the center to go. This method caused the stretching in the latitude to match that in the longitude. The meridians are straight vertical lines that are equally spaced. The parallels are straight horizontal lines but are unevenly spaced, becoming more separated nearer the pole. The poles are not on the map.

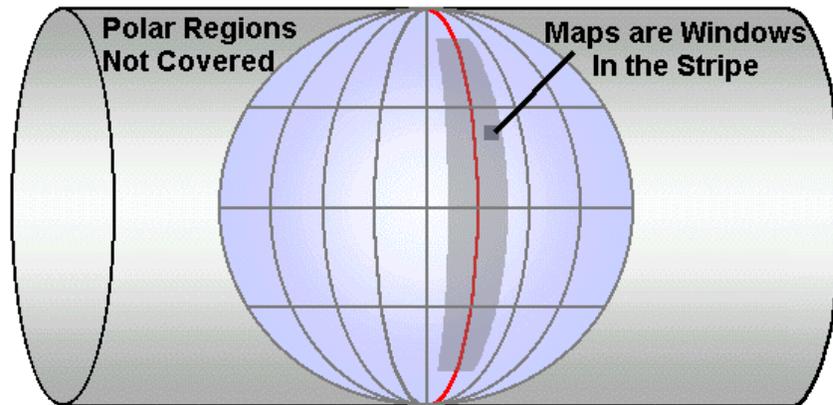
For the Mercator projection the distortion parameters are:

$$h = k = 1 / \cos \phi .$$

This projection is therefore conformal. But the product $h \cdot k$ is not 1, so it is not equal area.

B. Universal Transverse Mercator (UTM)

This is a Mercator based on a cylinder that is turned on its side so that the tangent line would be a meridian of longitude. In fact the cylinder is slightly depressed into the earth. The amount is defined by specifying the scale value on the central meridian to be 0.9996 . Because it is a Mercator projection, it is conformal. (Rotating the cylinder just rotates Tissot's Indicatrix, which is a circle and remains a circle if you rotate it.) The meridians are now curved lines. The parallels are straight lines. The meridians bow in. This means that azimuths will be in error for points not on the central meridian.



**Universal Transverse Mercator
Cylinder Turned on Side
Used in 6° Wide Stripe or Zone**

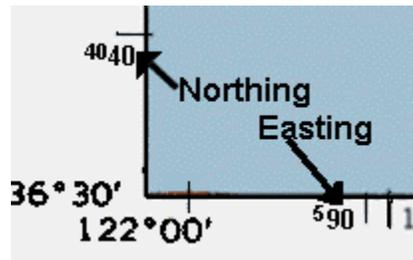
The scale error will be zero on the meridians that cut the earth. These are standard lines. Along the central meridian the scale factor is 0.996 and the scale error is 0.0004 (1 - 0.9996). This error grows as you go east or west of the standard lines. The maximum scale error is controlled by only using the projection +/- 3 degrees of longitude of the central meridian.

This is a zoned map. There are 60 separate projections, one for each 6 degree band of longitude beginning at 180E. This controls the error. The green line in the distortion graph above is in fact the maximum error for the UTM projection. The first zone covers 180E to 186 E, the second 186 E to 172 E and so forth. Technically the UTM only is defined from 84 N to 80 S.

This is a **windowed projection**. What is seen on a map, is a part of the projection for the whole zone. This means that adjacent maps have edges that line up, except where they are from different zones.

The coordinates in UTM are expressed in meters of **Northing and Easting**. The northing is the number of meters north of the equator. In the southern hemisphere this would normally be a negative number. However an offset of 10,000,000 (ten million) is added to northing values in the southern hemisphere. This offset value is called the **False Northing**. This offset keeps all values positive. The Easting is similarly defined. It is the number of meters east of the central meridian along a parallel with an offset of 500,000 (five hundred thousand) added. The offset is called the **False Easting**. Again it keeps the values positive (you can only go 3 degrees west of the central meridian before you enter another zone).

The values of easting and northing are often shown on topographic maps, even though the projection is not UTM. The locations of round values of easting and northing are computed and placed on the edges. On small area (large scale) maps these will be approximately uniformly spaced.



Here is the corner of a topographic map from that includes Monterey CA. The odd numbers are UTM tics. The small values are 100,000 values. Thus the northing value shown is 4,040,000 meters and the easting value 590,000 meters.

While the lines of easting and northing are perpendicular, because they are really parallels and meridians, they are not straight. At high accuracy you cannot use differences of easting and northing to find azimuths on a non-conformal projection. The values will be close for large scale maps, but not correct.

C. Transverse Mercator / State Plane Coordinate System

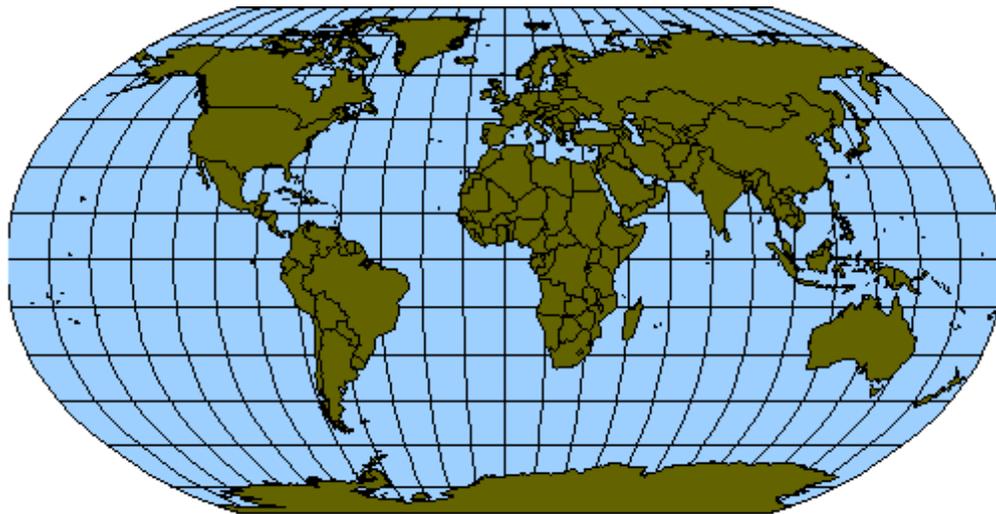
The United States has an official set of projections for legal land surveying. While state law defines this for each state, this was drawn together by the US Geological Survey as the **State Plane Coordinate System**. Each state has at least one area or **Zone**. Most states have 2 or three zones. Within the area of each zone there is one official, legal projection that must be used for filing maps with county and state recorders. There are about 115 zone in the US. All but one are either Transverse Mercator or Lambert Conic Conformal. The north-south type zones are TM and the east-west types zones are LCC. (The panhandle of

Alaska is an Oblique Mercator - that is a Mercator rotated so that it is transverse to the axis of the panhandle.)

The TM are not Universal Transverse Mercator because that term is restricted to the specific zone boundaries and central scale factor given above. In the State Plane Coordinate System the zones are usually smaller to limit the maximum error. The cylinder is depressed to give approximately the smallest maximum scale error over the zone. In practice zone boundaries are not straight but follow county boundaries.

D. Robinson

In the 1960's the Rand McNally map company wanted a new projection that would present the world well, that would "look nice". They wanted a projection that represented shapes well over the entire world. Arthur Robinson produced his projection for them. This projection had no mathematical close form representation, that is no functions f and g . It was defined by a table that showed the correspondence between (latitude, longitude) points on the earth and (x,y) map points. Today there are computer programs that have mathematical approximations that fit Robinson's table.

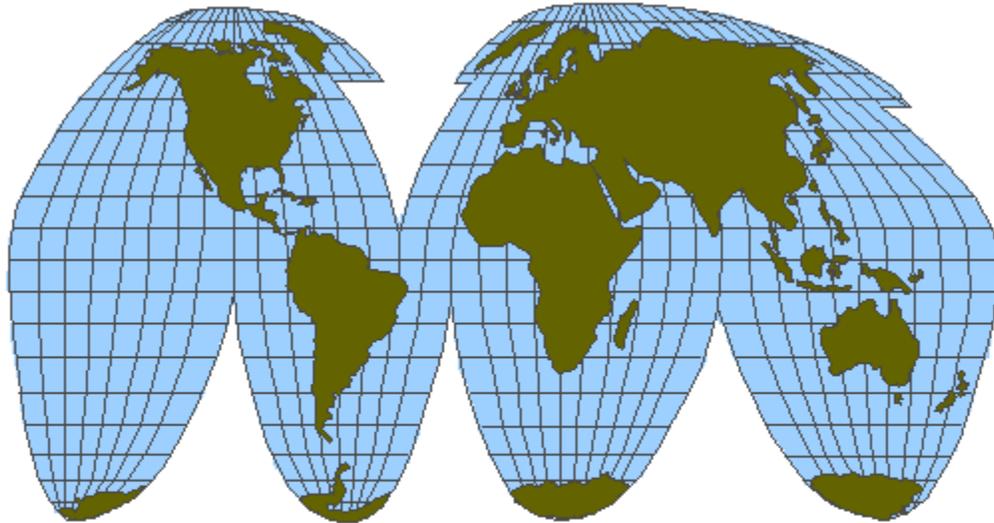


Robinson projection

This is a very common projection for world wide thematic material.

E. Goode

The presentation of thematic material on a world wide basis is also commonly done on the Goode projection. Officially this is an interrupted sinusoidal projection. But the two forms used by Goode have associated his name with the maps. He has one projection that puts the cuts in ocean areas.



Goode Projection (Interrupted Sinusoidal)

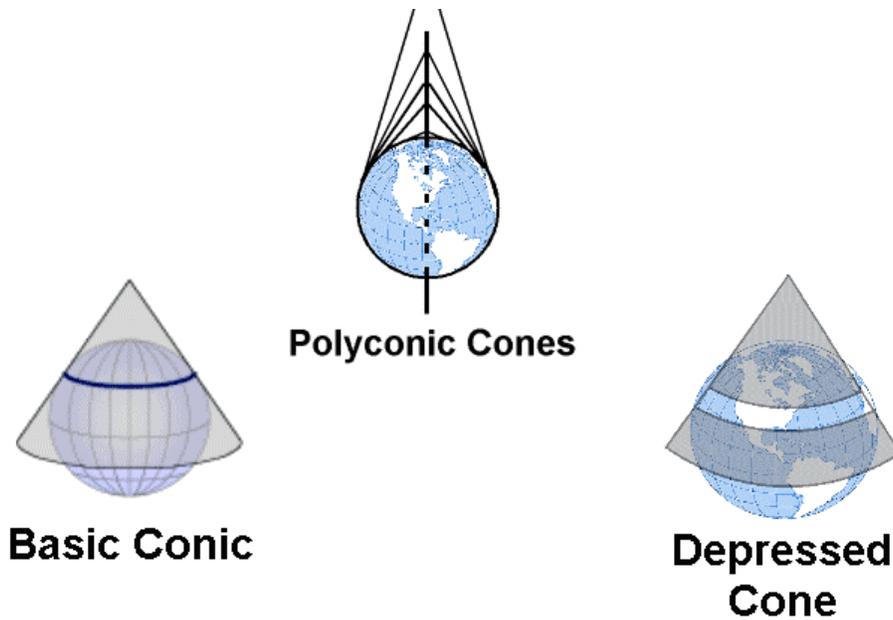
This is used to show many things, such as global temperature, forest area, population density etc. There is a second version that keeps the board ocean areas together and is use to display ocean properties.

IV Conic Projections

As with the other families of projections there are many conics. They basically have the same starting geometry, a cone over the earth, or slightly depressed into the earth. The real difference is mainly the way the latitudes are converted to the y-axis of the map.

The basic geometry of the different types of conics are shown below. In the basic conic, the cone is tangent to the world at some latitude. For error control several conics depress the cone into the earth generating two key parallels, where the cone crosses the earth surface. A variation was introduced in the 1800's by Hassler, the first superintendent of the US Geological Survey. This is the polyconic. The error increases in most projections as you go away from the point the surface touches the earth. Hassler had a new cone for each latitude, tangent at that latitude. This reduced the maximum scale error.

Conics do not do well representing the entire world. However they do very well on areas that are essentially east-west in orientation and less than a quarter of the world. The US fits this category. The smaller the area, the better the conic can do. The USGS topographic maps were in the polyconic projection from 1866 until 1957. After that some Lambert Conformal Conics were issued. These are maps that normally cover areas less than a degree in each direction.



Conic Projection Geometry

A. Lambert Conic Conformal

This is a simple conic that has been depressed into the earth. It is used for almost all aircraft navigation charts. The conformal nature of the projection is critical here. The map is normally only a small segment of the entire unrolled cone. The lines where the cone cuts the earth are standard lines, where the scale is true. These are usually about 10 percent of the way in from the top and bottom edges. These maps are usually about 2 to 3 times as wide as they are tall. (This is for the projection. Small windows may be published on individual maps. This makes the maps line up.)

B. Polyconic

The error in a conic increases away from the tangent point. Two or three cones merged together would reduce the error. Hassler went to the extreme and had a new cone for each latitude. While it might seem that this would lead to complex map making, the construction is really easy. In text books and on the internet there are examples of polyconics of the world. However in practice they are only used for very small areas. In this case the distortion is quite small. However the projection is neither conformal or equal area. But in the small area maps the angle errors are small.

C. Adler's Equal-Area (Conic)

This is common for non-navigation maps of the United States. It was used for most USGS maps at small scale (large area) in the past. Snyder recommends this for applications where an equal area map is desired. It is a conic projection, but the world conic is often left off the name. It has two standard parallels just like the Lambert.



Adler Equal-Area Conic
With Tissot's Indicatrix

V. References

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<http://www.usgs.gov/pubprod/>
3. Snyder, John P., *Map Projections, A Working Manual*, US Geological Survey Professional Paper 1365, 1987, US Government Printing Office.
4. *Datums, Ellipsoids, Grids, and Grid Reference Systems*. Edition 1. US National Imagery and Mapping Agency (NIMA). TM 8358.1, Washington, D.C., NIMA, 20 September 1990. (Available at reference 1 website as a PDF file.)
5. *The Universal Grids: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS)*. Edition 1. US National Imagery and Mapping Agency (NIMA). TM 8358.2 Washington, D.C., NIMA, 18 September 1989. (Available at reference 1 website as a PDF file.)